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AN EXPERIMENTAL STUDY OF SOUND TRANSMISSION FROM AIR
INTO BUBBLY WATER

Bruce Maccabee

Naval Surface Weapons Center
White Oak Laboratory, Silver Spring, Maryland

July 1975

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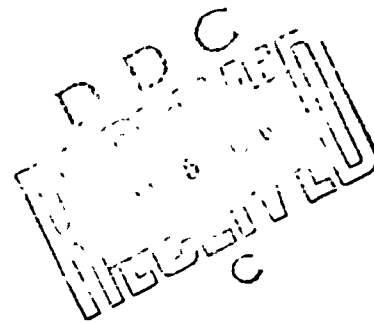
AN EXPERIMENTAL STUDY OF SOUND TRANSMISSION FROM AIR INTO BUBBLY WATER

BY
Bruce Maccabee

JULY 1975

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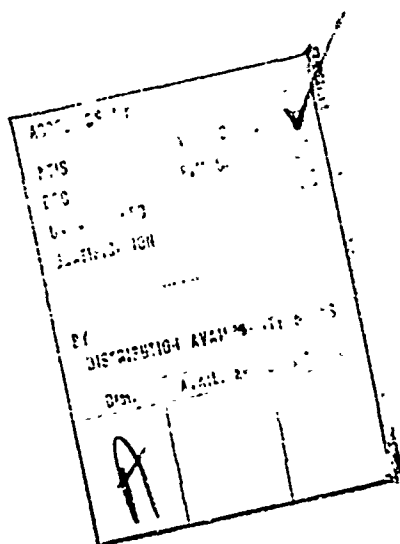
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have been observed in laboratory experiments in a simple hydro-acoustic tank (not acoustically damped). In these experiments, sound pressures measured near the apex of a conical shaped bubbly region (apex at its bottom) have been found to be as much as two orders of magnitude greater, at certain frequencies, than pressures measured at the same place in the tank in the absence of bubbles. To account for the anomalously high pressure increases in the presence of the conical bubbly region, it is proposed that the region acts as a "leaky" waveguide which "traps" sound rays incident on the surface and concentrates the rays as they travel downward toward the apex of the cone. It is proposed that these experiments be repeated in an acoustically damped tank.



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AN EXPERIMENTAL STUDY OF SOUND TRANSMISSION FROM AIR INTO BUBBLY WATER

The work reported here is a preliminary experimental study of the effects of dense populations of tiny bubbles on the propagation of low frequency sound (less than 5 KHz) from air into water. Under certain conditions the underwater sound pressure due to a source in the air can be as much as several orders of magnitude greater when a column of bubbles is present than it is when there are no bubbles. The increased underwater sound pressure in the presence of the bubbles depends upon the bubble density (fractional bubble volume) and probably also on the spatial distribution of the bubbles. The experiments were complicated by the presence of naturally occurring resonance modes of the hydroacoustic tanks and by non-resonant scattered and reflected energy from the tank walls. These complications raise some doubts about the validity of the experimental results. Therefore, in order to eliminate the effects of energy reflected from the tank walls and to thereby remove all doubts as to the effects of the bubbles, it is recommended that further work be done in very large or acoustically damped tanks.

The author thanks Noreen Prochaska and George Boyers for their support of this research, which was sponsored by the Office of Naval Research under ONR Task 75-WR-50247.



LEMMUEL L. HILL
By direction

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I. INTRODUCTION

A preliminary experimental study of the effects of large densities of small bubbles on the transmission of sound from air into water has been carried out. The investigation has centered around the transmission of sound from a source in air (a loud-speaker) to a hydrophone in a water tank without and then with the presence of a vertical or upward-curving conical column of small bubbles. The bubble columns have been generated by the forced flow of bubbly water from a small circular port into the acoustic tank. Although fundamentally straightforward, the experiment has been complicated by the presence of many resonance modes in the water tanks that have been used, and by the non-uniform bubble distributions that have been produced in the tanks. Consequently the experimental results to be presented are mainly qualitative.

II. EXPECTATIONS FROM SIMPLIFIED THEORY

The basic physical conditions of the experiments considered here are that 1) the volume fraction, β = volume of bubbles/total volume, of bubbles in water is very large ($> 10^{-4}$) compared to normally occurring fractions ($< 10^{-5}$), and 2) the bubbles present in the population do not resonate at the acoustic test frequencies (0-5 KHz). In most papers the investigators have considered frequencies greater than about 5 KHz, to which the larger of normally occurring or man-made bubbles are resonant or nearly resonant¹⁻⁵. The resonant frequency of a bubble depends upon its radius and upon the surrounding water pressure, but is essentially independent of temperature at normal environmental temperatures. For bubbles within several meters of the water surface the resonant frequency and wavelength are given by⁶

$$f = (326/r) \text{ in Hz} \quad (1A)$$

$$\lambda = 152000/f = 466 r \text{ in cm} \quad (1B)$$

where r is in cm and the speed of sound (at about 20° C) is set equal to 1.52×10^5 cm/sec. Figure 1 illustrates these equations for various values of r . The resonance characteristics of a bubble are given by⁵

$$\sigma/\sigma_r = \delta^2 / [(f_r^2/f^2 - 1)^2 + \delta^2] \quad (2)$$

where σ is a cross-section (for scattering or absorption) at frequency f , σ_r is the value of the cross-section at the resonant frequency, f_r , and δ is the damping constant. Since the damping constant is generally less than⁶ 0.1, Equation (2) corresponds to a

¹E.L. Carstensen and L.L. Foldy, J.A.S.A. 19, 481 (1947)

²V.P. Glotov, P.A. Kolobaev, and G.C. Neuimim, Sov. Phys.-Acoustics 7, 341 (1962)

³B.S. McCartney and B. McK. Bary, Deep Sea Res. 12, 285 (1965)

⁴C.S. Clay and H. Medwin, J.A.S.A. 36, 2132 (1964)

⁵H. Medwin, J. Geophys. Res. 75, 599 (1970)

⁶C. Devin, Jr., J.A.S.A. 31, 1654 (1959)

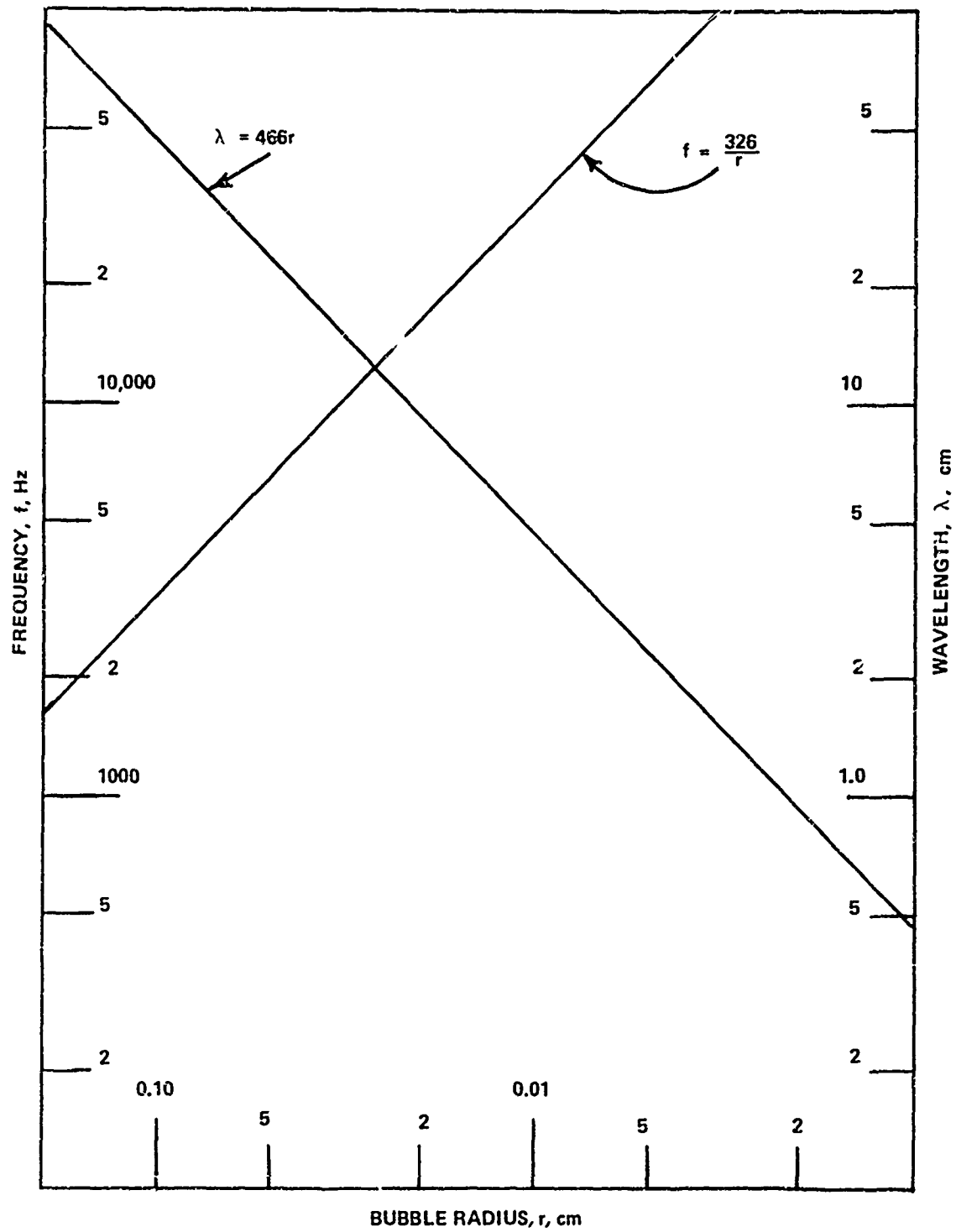


FIG. 1 ACOUSTIC RESONANCE FREQUENCIES OF INDIVIDUAL BUBBLES

rather sharp resonance characteristic. For example, at $f_r/f = 1/2, \sigma/\sigma_r \approx 0.017$ when $\delta = 0.1$. Normally occurring bubble populations, in which the bubbles are small enough to persist underwater for many seconds as they rise to the top, consist of bubbles with radii less than about 500 microns, so the normally occurring resonant frequencies are greater than about 6 KHz (see Figure 1). Thus, to avoid resonance effects such as scattering and absorption due to the bubbles it is necessary to use frequencies lower than 6 KHz. Laird and Kendig⁷, during an investigation of bubble screen attenuation, extended their measurements to frequencies below 6 KHz. They found very little attenuation of low frequency sound (~ 2 KHz) that passed through a rather dense ($\beta \sim 5 \times 10^{-4}$) bubble screen that attenuated resonant frequency sound waves ($f_r \sim 9$ KHz) by as much as 15 db/inch.

Theoretical treatments of the effects of bubbles in the long wavelength limit have been presented. Wood⁸ showed that to calculate the speed of sound in the long wavelength limit, bubbly water could be treated as a homogeneous mixture having a density and compressibility that are each intermediate between the densities and compressibilities of "pure" air and water. Urick^{9,10} used this theory in an experimental determination of compressibilities of substances in suspension, but not of bubbles. Hsieh and Plesset¹¹ have placed the Wood theory on a more firm theoretical foundation and have shown clearly why it is the isothermal compressibility and not the adiabatic compressibility which enters the calculation of the speed of sound in the bubbly medium. They also showed that the attenuation of sound in bubbly water in the long wavelength limit is much smaller than the attenuation in air alone.

The speed of sound in bubbly water at frequencies considerably below the lowest resonant frequencies of the bubbles (a distribution of bubble sizes is assumed) is given by^{8,9,10,11}

$$v = \left\{ \left[\rho_1 \beta + \rho_2 (1-\beta) \right] \left[f K_1 + (1-\beta) K_2 \right] \right\}^{-1/2} \quad (3A)$$

⁷D.T. Laird and P.M. Kendig, J.A.S.A. 24, 29 (1952)

⁸A.B. Wood, A Textbook of Sound, G. Bell and Sons, Ltd., London (1941)

⁹R.J. Urick, J. Appl. Phys. 18, 983 (1947)

¹⁰R.J. Urick, Principles of Underwater Sound for Engineers, McGraw-Hill Inc. New York, N.Y. (1967)

¹¹D. Hsieh and M.S. Plesset, J. Phys, Fluids 4, 970 (1961)

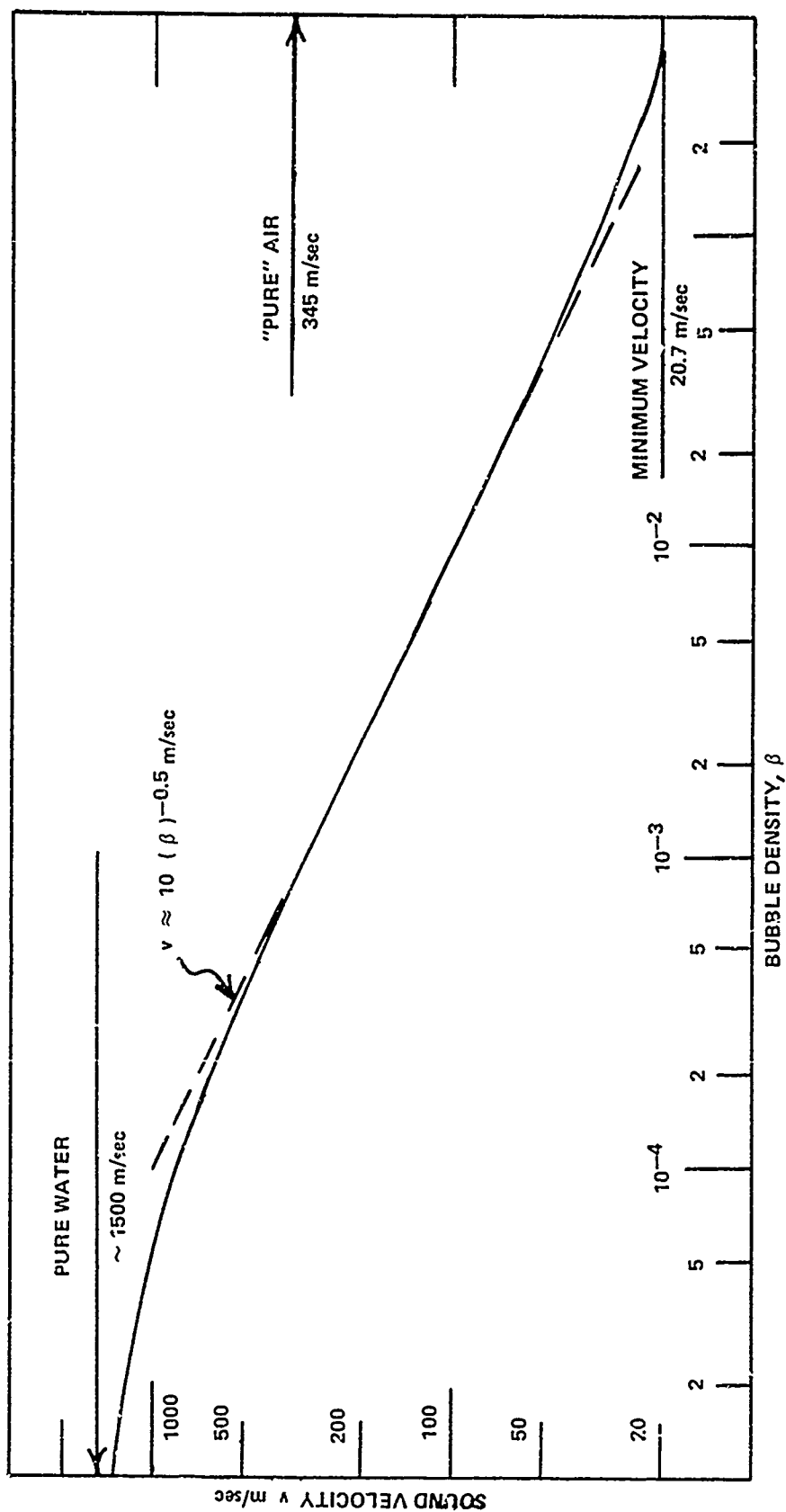


FIG. 2 SOUND VELOCITY IN BUBBLY WATER

where ρ_1 is the air density, ρ_2 is the water density, K_1 is the isothermal compressibility of air, and K_2 is the isothermal compressibility of water. (Note: in non-bubbly water the sound speed is given by $v = (\rho K)^{-1/2}$ where K is the adiabatic compressibility; for air $K = K_1/1.4$ and for water $K = K_2/1.05$, so velocities calculated using the isothermal compressibilities will not differ greatly from those calculated using adiabatic compressibilities.) Thus the velocity of the mixture is a function of the average compressibility and the average density. By factoring out the (isothermal) speed of sound in bubble-free water, and by correcting that speed to obtain the (adiabatic) speed of sound in bubble free water (multiply the isothermal speed by $(1.05)^{1/2}$), Equation (3A) can be written as follows:

$$v = v_p = 0 \left[(\beta \rho_1 / \rho_2 + 1 - \beta) (\beta K_1 / K_2 + 1 - \beta) \right]^{-1/2} \quad (3B)$$

$$= 1520 \left[(1 - \beta) (1 + 21500\beta) \right]^{-1/2} \text{ in m/sec.} \quad (3C)$$

where in Equation (3C), $(\rho_1/\rho_2)\beta = 0.0012\beta$ has been ignored compared to $1 - \beta$, and β has been ignored compared to $1 + \beta(K_1/K_2)$. Equation (3C) is accurate for β as large as 0.95. Figure 2 illustrates the variation of sound velocity with the bubble fraction. The quadratic dependence on β accounts for the existence of a minimum velocity (~ 21 m/sec.) which occurs at $\beta = 0.5$. (The speed of sound in air has been set at 345 m/sec.) The log-log plot conveniently "stretches out" the region of interest: $\beta < 0.5$.

Of particular interest in this research is the transition of sound waves from air into bubbly water (and then into bubble-free water). The transmission through an interface is governed by the relative acoustic impedances of the media at the interface (and by interference phenomena in the event that there are several parallel interfaces separated by multiples of the sound wavelengths in the media). The acoustic impedance, ρv , of a bubbly medium as a function of β is illustrated in Figure 3. For $\beta < 10^{-1}$ the impedance is very nearly that of pure water.

The transmitted intensity ratio, α_t , for a plane wave going from one semi-infinite medium to another is given by¹²

$$\alpha_t = \frac{4\rho_1 v_1 \rho_2 v_2 \cos^2 \theta_1}{(\rho_2 v_2 \cos \theta_1 + \rho_1 v_1 \cos \theta_2)^2} \quad (4)$$

where θ_1 is the angle of incidence and θ_2 is the angle of refraction. These angles are related by Snell's law:

¹²K.E. Kinsler and A.R. Frey, Fundamentals of Acoustics, J. Wiley and Sons, New York, N.Y. (1962)

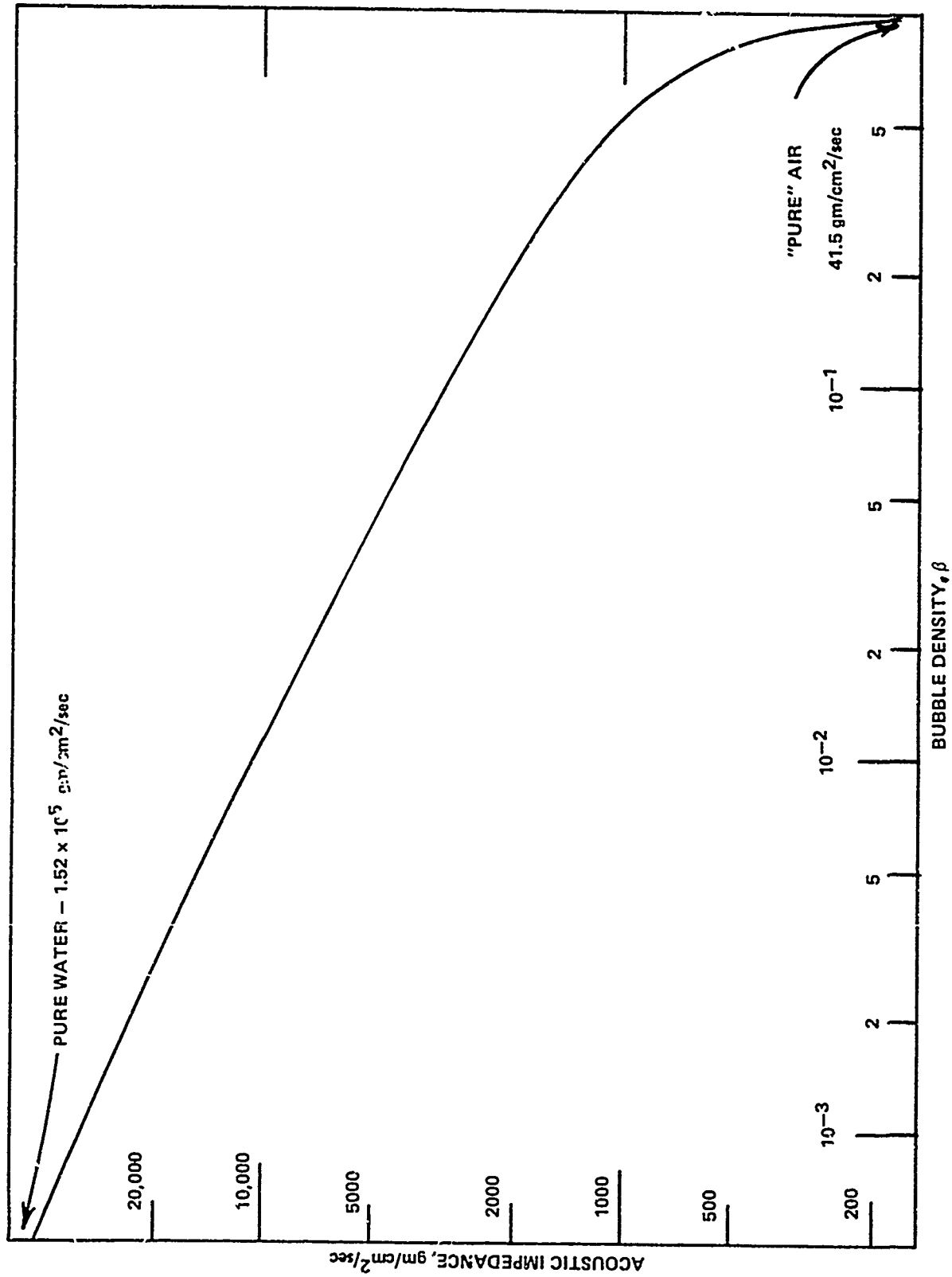


FIG. 3 THE ACOUSTIC IMPEDANCE OF BUBBLY WATER

$$\sin\theta_1/v_1 = \sin\theta_2/v_2 \quad (5)$$

The transmitted pressure at the interface is¹³

$$p_2/p_1 = 1 + \frac{\rho_2 v_2 / \cos\theta_2 - \rho_1 v_1 / \cos\theta_1}{\rho_2 v_2 / \cos\theta_2 + \rho_1 v_1 / \cos\theta_1} \quad (6)$$

For the case of air ($\rho_1 v_1$) to bubble-free water ($\rho_2 v_2$), Equation (5) shows that sound incident at angles up to about 13° (the "critical" angle) will be transmitted through the interface into the water. For angles of incidence greater than about 13° the angle of refraction is "greater than 90° ", so the sound is reflected at the interface. However, the presence of bubbles decreases the speed of sound in the water so the critical angle is increased as illustrated in Figure 4. For β only as large as about 10^{-3} sound is transmitted from air into bubbly water at all incident angles (except the "grazing angle", $= 90^\circ$).

Since $\rho_2 v_2 \gg \rho_1 v_1$ even with β as large as 0.5, Equation (4) can be simplified to read

$$\alpha_t = 4\rho_1 v_1 / \rho_2 v_2 \quad (7)$$

For transmission from air ($\rho_1 v_1 = 41.5 \text{ gm/cm}^2/\text{sec.}$) into bubble-free water ($\rho_2 v_2 = 152000 \text{ gm/cm}^2/\text{sec.}$), α_t is about 0.0011. The presence of bubbles increases the transmission of acoustic intensity considerably, as illustrated in Figure 5.

Of more direct interest from the experimental point of view is the dependence of the transmitted pressure on β , because the measurements are made with hydrophones. As long as $\rho_2 v_2 \gg \rho_1 v_1$, Equation (6) can be simplified to $p_2/p_1 = 1+1 = 2$. That is, the pressure in the water at the interface is twice that in air at the interface. Thus, for a plane wave incident on bubbly water for all reasonable values of β ($\beta \approx 0.5$) the transition causes an "amplification" (doubling) of the pressure that is essentially insensitive to the bubble concentration. However, the transmission of a spherical wavefront radiated from a point source in air is sensitive to β . Ray calculations show that when the receiver is directly under the source^{14,15}

¹³H. Medwin and J.D. Hagy, Jr., J.A.S.A. 51, 1083 (1972)

¹⁴R.W. Young, J.A.S.A. 50, 1392 (1971)

¹⁵R.W. Young, J.A.S.A. 53, 1768 (1973)

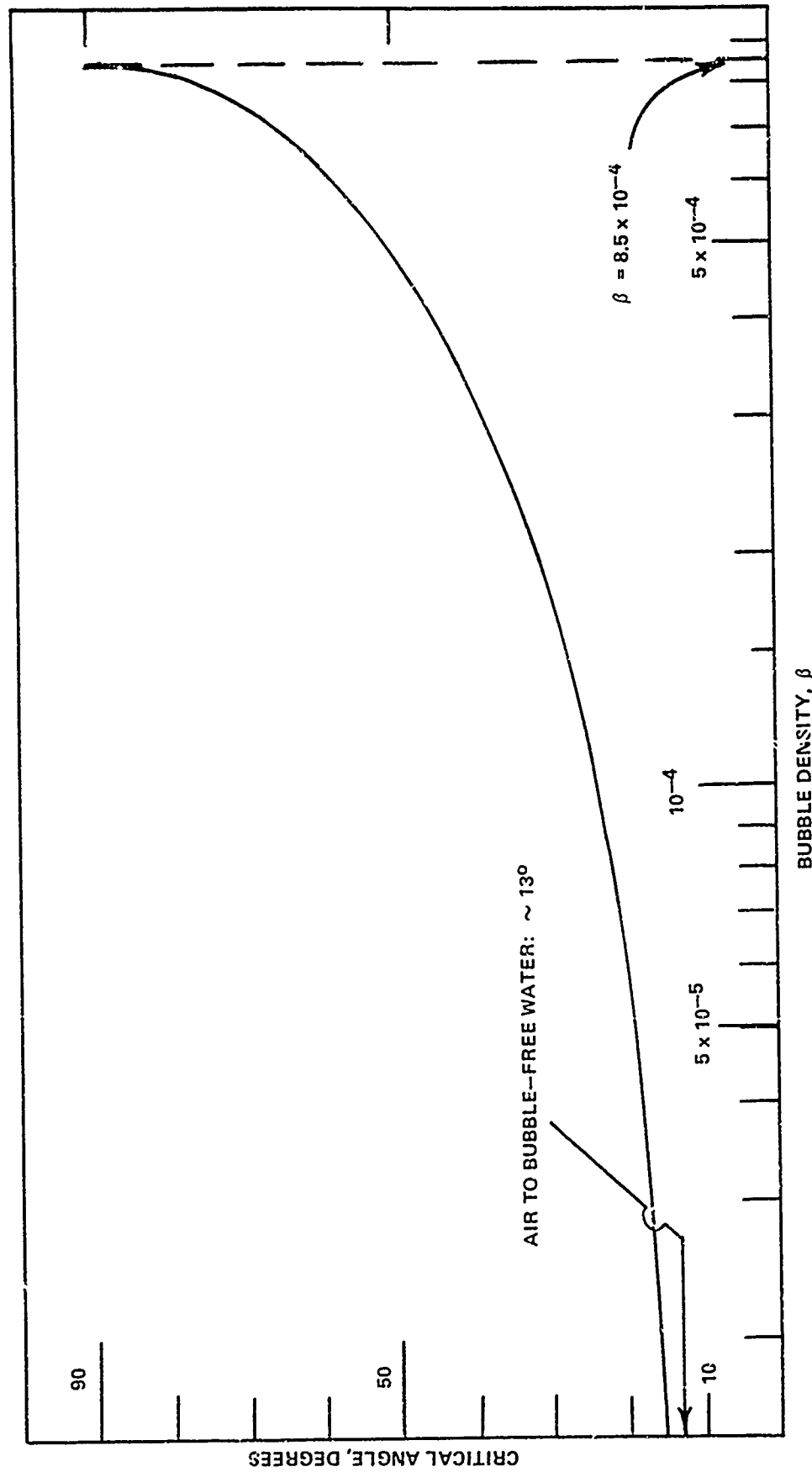


FIG. 4 THE CRITICAL ANGLE FOR SOUND TRANSMISSION FROM AIR INTO BUBBLY WATER

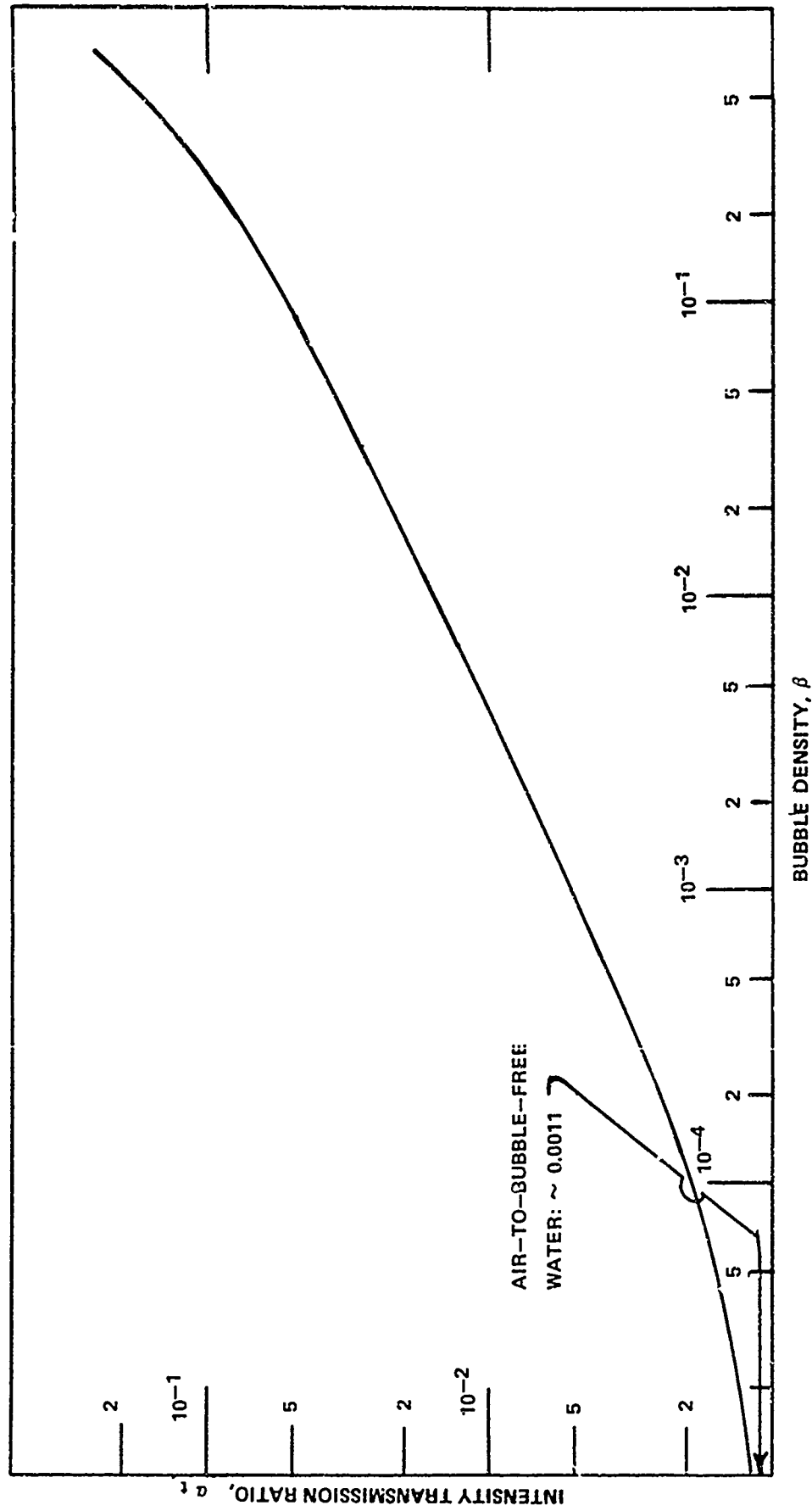


FIG. 5 THE AIR-TO-BUBBLY WATER SOUND INTENSITY TRANSMISSION RATIO (PLANE WAVE)

$$p(z) = \frac{p_s r_s (2 v_1/v_2)}{(h v_1/v_2 + z) \left[1 + (\rho_1 v_1)/(\rho_2 v_2) \right]} \quad (8)$$

where p_s is a reference pressure, r_s is a reference distance (assumed to be in air) from the source, h is the height of the source above the (water) surface, and z is the depth below the surface. The denominator essentially represents the distance from the receiver at depth z to a "virtual source" at height $(v_1/v_2) h$; the pressure of the virtual source is $p_s r_s (2v_1/v_2)$. The factor 2 results from the doubling of the pressure at the interface, as previously described. Equation (8) can be rewritten in a manner that is convenient for graphical presentation:

$$\frac{p(z)^2}{p_s^2 r_s^2} = \frac{2}{h/z + v_2/v_1} \times \frac{1}{(1 + \rho_1 v_1/\rho_2 v_2)} \quad (9)$$

For constant values of z , h , and $p_s r_s$, $p(z)$ varies with β as illustrated in Figure 6, which demonstrates the variation for various ratios h/z . Figure 6 shows that the pressure at the receiver is quite sensitive to the presence of bubbles. For $h/z = 0.1$ and with β only about 1.4×10^{-4} the transmitted pressure is twice what it is when $\beta = 0$; for $\beta = 10^{-3}$ the pressure is about 5 times its value when $\beta = 0$.

Equation (9) predicts that the maximum pressure transmission occurs for $h/z = 0$ when $\beta = 0.5$ (see Figure 6). The pressure increase factor is about 32 (30 db SPL increase). This value represents an expected upper bound to the attainable pressure increase due to bubbles in the geometrical arrangement represented by Equation (8). Any real case will have a lower factor because 1) $h \neq 0$, 2) experimental values of β as large as 0.5 probably cannot be obtained, even in soapy water, and 3) a real source is not a perfect point source.

In the more general case when the source is not directly above the receiver Equation (8) becomes very complicated. However, the general result that the pressure in the bubbly water due to a very small (point) source in air increases with β is still valid.

Thus far only the case of transmission from air into a bubbly water medium has been considered. If the bubbly water forms a layer of some definite thickness (depth) at the surface of the body of water, and if the hydrophone is below the layer (i.e., in bubble-free water) the situation is more complicated still. If a plane wave is incident normally on an "ideal" bubble layer of uniform bubble density (β is constant throughout the layer), and if the layer is an odd multiple of quarter waves in thickness at the sound frequency being transmitted, then interference can take place in

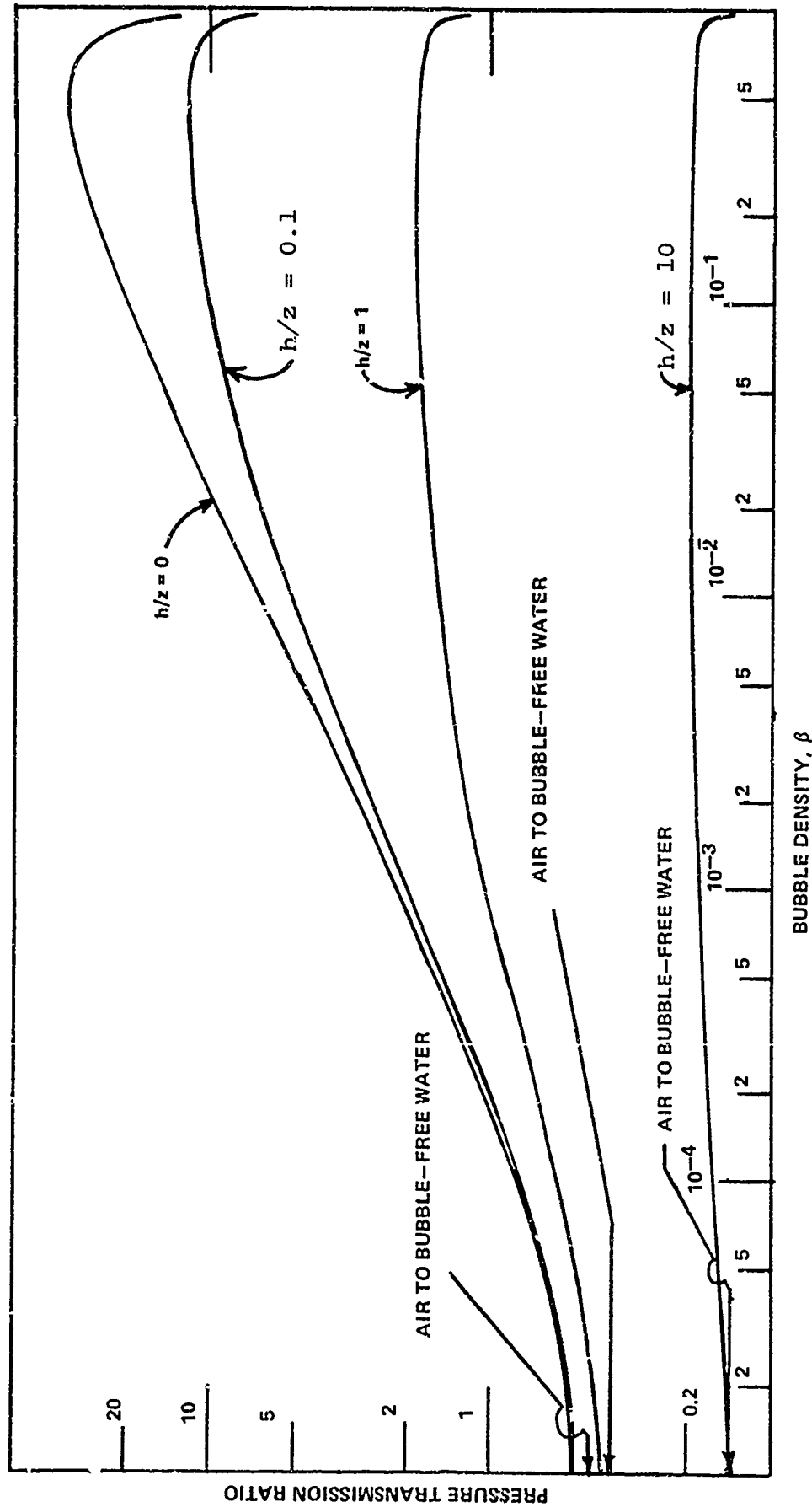


FIG. 6 PRESSURE TRANSMISSION FROM A POINT SOURCE IN AIR AT HEIGHT h TO A RECEIVER IN BUBBLY WATER AT DEPTH z

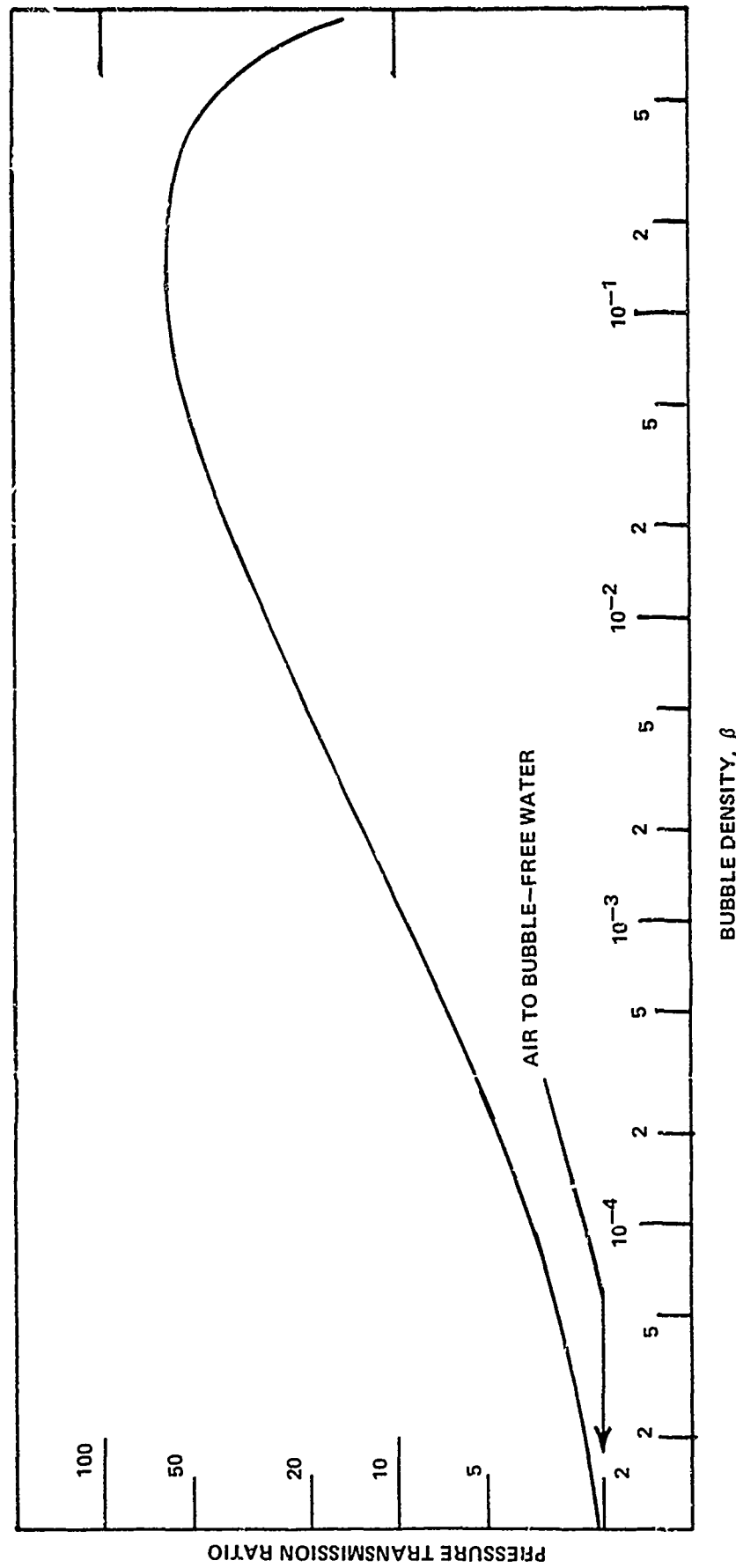


FIG. 7 PRESSURE TRANSMISSION OF A NORMALLY INCIDENT PLANE WAVE FROM AIR TO BUBBLE-FREE WATER VIA A LAYER OF BUBBLY WATER OF THICKNESS $(2n - 1) \lambda / 4$; (NO ABSORPTION)

such a manner that the transmitted intensity equals the incident intensity,¹² provided that the acoustic impedance of the intervening bubble layer is the geometric mean between the impedances of air and water: (ρv) bubble layer = $(1.52 \times 10^5 \times 41.5)^{1/2} = 2511 \text{ gm/cm}^2/\text{sec}$. In this case the pressure ratio is the square root of the impedance ratio: $p_3/p_1 = 60.5$, where p_3 is the pressure in the bubble-free water and p_1 is the pressure in the air. If the bubble layer impedance is not the geometric mean, the transmitted intensity ratio is not unity. Equation (10) expresses the variation of the pressure ratio with the impedance of the intermediate layer (the subscript 2 refers to the bubble layer):

$$p_3/p_1 = 2\rho_3 v_3 \rho_2 v_2 \left[(\rho_2 v_2)^2 + \rho_1 v_1 \rho_3 v_3 \right]^{-1} \quad (10)$$

Figure 7 illustrates the variation of the pressure transmission ratio with β . The peak pressure transmission ratio, 60.5, is reached at $\beta = 0.145$. Clearly the bubble layer matches the impedance of the air to the impedance of the water. However, in an experimentally realizable situation the matching that is afforded by a homogeneous bubble layer may not be obtained because the bubble density varies with height throughout the layer. Nevertheless, it seems reasonable to assume that for any real bubble layer there is an equivalent or nearly equivalent "ideal" bubble layer which effects some degree of impedance matching at a frequency for which the wavelength is at least commensurate with four times the thickness of the real bubble layer. A consequence of this assumption is that any real bubble layer will transmit certain frequencies or bands of frequencies better than other frequencies. (Note: at frequencies above about 6 KHz, bubble attenuation due to absorption and scattering is expected to become quite important. The interaction between impedance matching effects and attenuation effects at these "high" frequencies are not considered here.) For example, assume the equivalent homogeneous bubble layer is 50 cm thick. The frequency for which $50 \text{ cm} = \lambda/4$ corresponds to $\lambda = 2 \text{ m}$. To determine the frequency it is necessary to know the sound velocity in the bubble layer. This, in turn, requires knowledge of the bubble fraction. Assume $\beta = 10^{-3}$. From Figure 2 the velocity is about 320 m/sec. (i.e., almost the same velocity as in air), so the lowest frequency that will be transmitted is $f = v/\lambda = 160 \text{ Hz}$. Frequencies of $(2n-1) \times 160 \text{ Hz}$ will also be transmitted. The pressure transmission through the layer into the bubble-free water, from Figure 7, is then about 4 times greater than from air directly into bubble-free water.

For plane waves incident at angles very close to normal incidence the pressure transmission given by Figure 7 is approximately correct. However, at angles greater than about 5° the

expression for the transmission through a plane layer becomes quite complicated, so Figure 7 is no longer a valid indicator of the transmission ratio.

The general conclusion from the preceding simple theoretical analysis is that the transmitted pressure should depend in some (complicated) way upon the bubble density and, in general, should increase with that density. Exact solutions to "ideal" problems (plane wave source, point source, homogeneous layer, etc.) are instructive but are difficult, if not impossible, to relate quantitatively to realizable experimental situations. In particular, in the experiments reported here, the bubble distributions have not been homogeneous either vertically or horizontally, and the finite sized containers have introduced resonances into the spectrum of the sound transmitted into the water. Because of the complexity introduced by the experimental arrangements, probably the main things worth remembering from the previous analysis are 1) the sound velocity does not change much for $\beta < 10^{-4}$, so effects of dense bubble populations will probably only be measurable for $\beta > 10^{-4}$; 2) for $\beta > 10^{-3}$ sound at "all angles of incidence can be refracted into the bubbly water; 3) in a situation which is most nearly like that of a plane wave normally incident on a plane bubbly layer the sound pressure in the layer is only doubled, whereas the pressure in a spherical wave incident on the same layer may be increased by as much as an order of magnitude, but no more than about a factor of 30; 4) a plane wave incident normally on a plane bubbly layer of finite thickness will be transmitted into the water below the bubbly layer by an amount which depends upon the bubble density and upon the vertical thickness of the layer compared with the wavelength of sound in the bubbly water (frequency dependence); and 5), if all the sound intensity is transmitted into the water (perfect impedance matching), the acoustic pressure in the water will be about 60 times what it would be with no impedance matching. As will be seen the experimental results are in qualitative agreement with the predictions of the previous simple theory even though the experimental arrangements are quite different from the model arrangements. However, to explain the quantitative experimental results a much more complex theory will be necessary.

III. EXPERIMENTAL APPROACH AND APPARATUS

Investigations^{2,3,4,5,16,17,18} of naturally occurring bubble populations in oceans and lakes as well as in laboratory experiments have shown that, in general, the bubble fraction near the surface is so low ($\beta < 10^{-7}$) that low frequency sound is essentially not affected by the bubbles. However, Medwin¹⁹ has recently shown that in moderate seas the bubble population can affect the speed of sound by several tenths of a percent ($\beta \lesssim 10^{-7}$). These naturally occurring bubbles are generated by mechanical action (waves and whitecaps) at the surface, are carried beneath the surface by wave action, and either rise to the surface or dissolve underwater, so that the bubble population does not build up beyond a "steady state" value which is some function of the mechanical action of the waves. Natural trapping mechanisms (surface scum, biological gas entrapment), if they exist, could create somewhat larger bubble fractions within short distances (centimeters to meters) from the surface. Of course boats and other "unusual" phenomena (hurricanes, waterspouts, etc.) can create considerable bubble densities for short periods of time. Under such conditions β might become "momentarily" large ($\beta \lesssim 10^{-3}$), in which case sound transmission into the water could be somewhat enhanced.

Since normally occurring bubble densities are so small that the effects predicted by the preceding theory would be difficult to measure, the laboratory study of the effects of bubbles on sound transmission have centered around the effects of populations that are quite dense. Populations (β in the range 10^{-4} to 10^{-2}) of very tiny bubbles ($< 1/2$ mm) have been produced in the laboratory tanks by allowing the impeller of a pump to mix varying amounts of air and soapy water. In soap free water the pump will also produce bubbles that range from ~ 1 mm in diameter downwards, but these rise quickly to the surface. However, as soap is added to the water the mean bubble size shrinks while the rise time to the surface increases considerably.

¹⁶D.C. Blanchard and A.H. Woodcock, *Tellus* 9, 145 (1957)

¹⁷E.C. Monahan and C.R. Zeitlow, *J. Geophys. Res.* 74, 6961 (1969)

¹⁸J. Kanwisher, *Deep Sea. Res.* 10, 195 (1963)

¹⁹H. Medwin, "Sound Speed Dispersion and Fluctuations in the Upper Ocean: Project Bass", NPS-61Md73101A, Naval Postgraduate School, Monterey, Calif. (1973)

The general experimental arrangement is illustrated in Figures 8 and 9. Figure 8 shows the typical acoustic tank setup with one or more moveable hydrophones, a moveable bubble source, and a moveable acoustic source (speaker). Figure 9 illustrates the electronics involved in obtaining data on the transmission of sound from air into water. The experiments which have been done do not yield absolute measurements of the sound pressure transmission ratio $P_{\text{water}}/P_{\text{air}}$, since only the acoustic pressure in the water is measured. Instead, the pressure in the water is measured for a particular arrangement of the speaker and hydrophone(s) first without and then with bubbles in the water. Since the pressure in the air is constant - the drive to the speaker is monitored and kept constant - these measurements can be used to obtain the relative sound pressure increase, called the signal gain factor (SGF) in the presence of bubbles: $\text{SGF} = p_{w,b}/p_w$, where $p_{w,b}$ stands for the pressure in the water in the presence of bubbles. The spectral distribution of the effect of bubbles is obtained by sweeping an oscillator through the desired spectral range and recording the pressure on an X-Y recorder. For measurements of the change in sound pressure over a band of frequencies a white noise generator followed by an adjustable bandwidth filter is used as the sound source.

The acoustic tanks used were of two sizes. The small tank was roughly cubical with the length and width approximately 56 cm and the height of the water was about 52 cm. The large tank was about 153 cm long by 90 cm wide and was typically filled to a depth of 60 - 80 cm. Each tank was lined with about 10 cm of synthetic horsehair packing material to act as acoustic damping. It was found that when the packing material was "fresh", i.e., had just been placed in water for the first time, it effected a good degree of damping for a short time. However, over a period of several days or so the damping tendency diminished greatly. Eventually it was found that the damping caused by foam "rubber" packing material lasted long enough for experiments to be performed, but it, too, was not permanent.

The reason for attempting to damp the acoustic waves in the tank was that the spectrum of sound transmission from air into the tank was very complicated even in the absence of bubbles. Consequently a major problem of the investigation became the separation of the naturally occurring mode spectrum of the tank from whatever mode spectrum might be characteristic of the bubbles. Initial experiments in the small tank showed an increased sound transmission in the presence of bubbles in a spectral band centered around 1800 Hz. However, the small tank also had a large natural resonance mode at about 2200 Hz. (This frequency corresponds to a sound wave reflected by the bottom of the tank and the water surface; $\lambda = (52 \text{ cm}) / (3/4) = 69.3 \text{ cm}$ which corresponds to a frequency of $(1500 \text{ m/sec}) / (0.693 \text{ m}) = 2164 \text{ Hz}$. The lowest order vertical reflection mode of wavelength $(52 \text{ cm}) / (1/4) = 208 \text{ cm}$ was not observed.) It was felt that this natural resonance frequency might be "pulled" downward several hundred hertz and broadened by the presence of the bubbles thus making it appear that the bubbles had enhanced

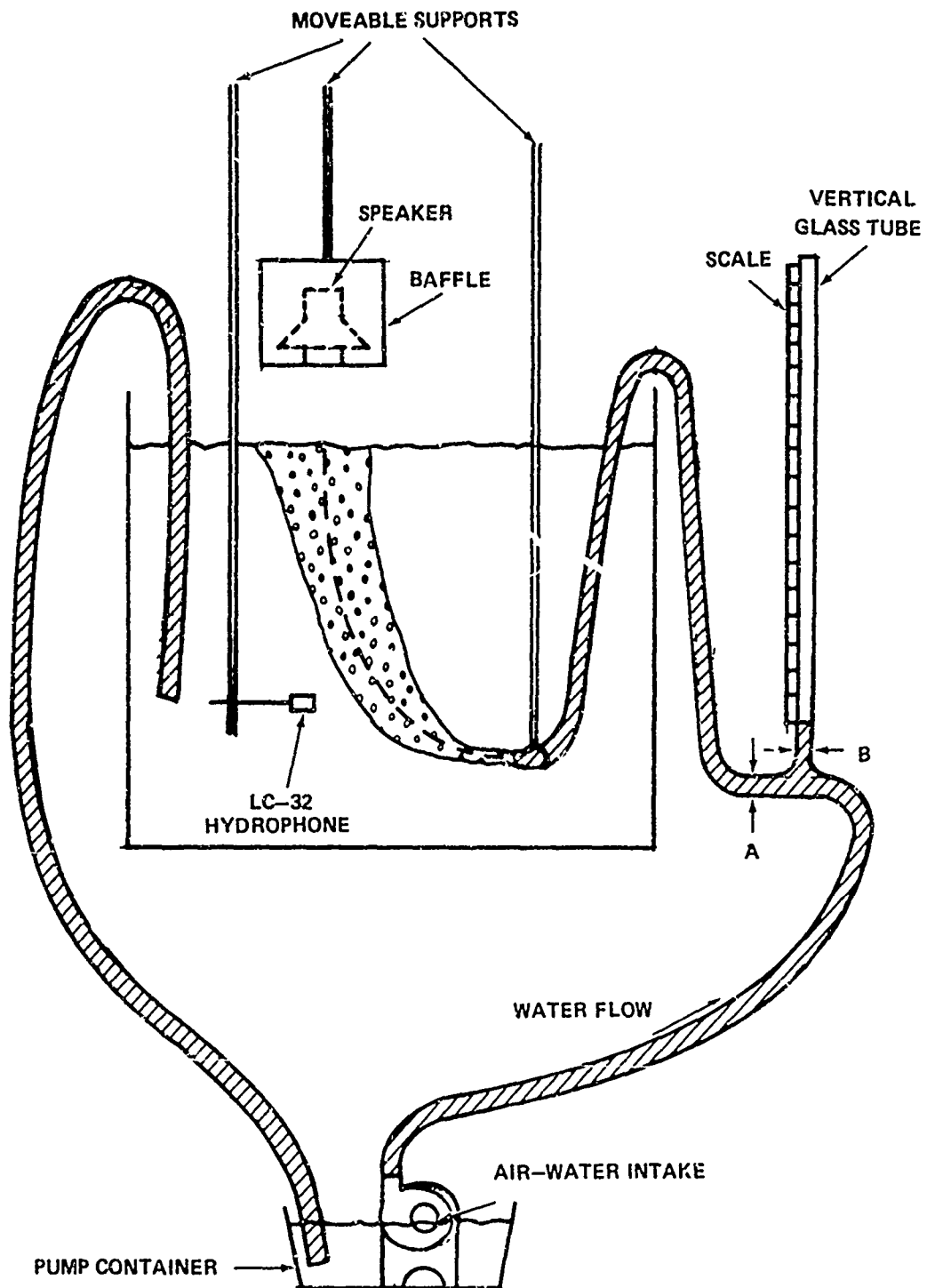


FIG. 8 THE BASIC ACOUSTIC TANK ARRANGEMENT

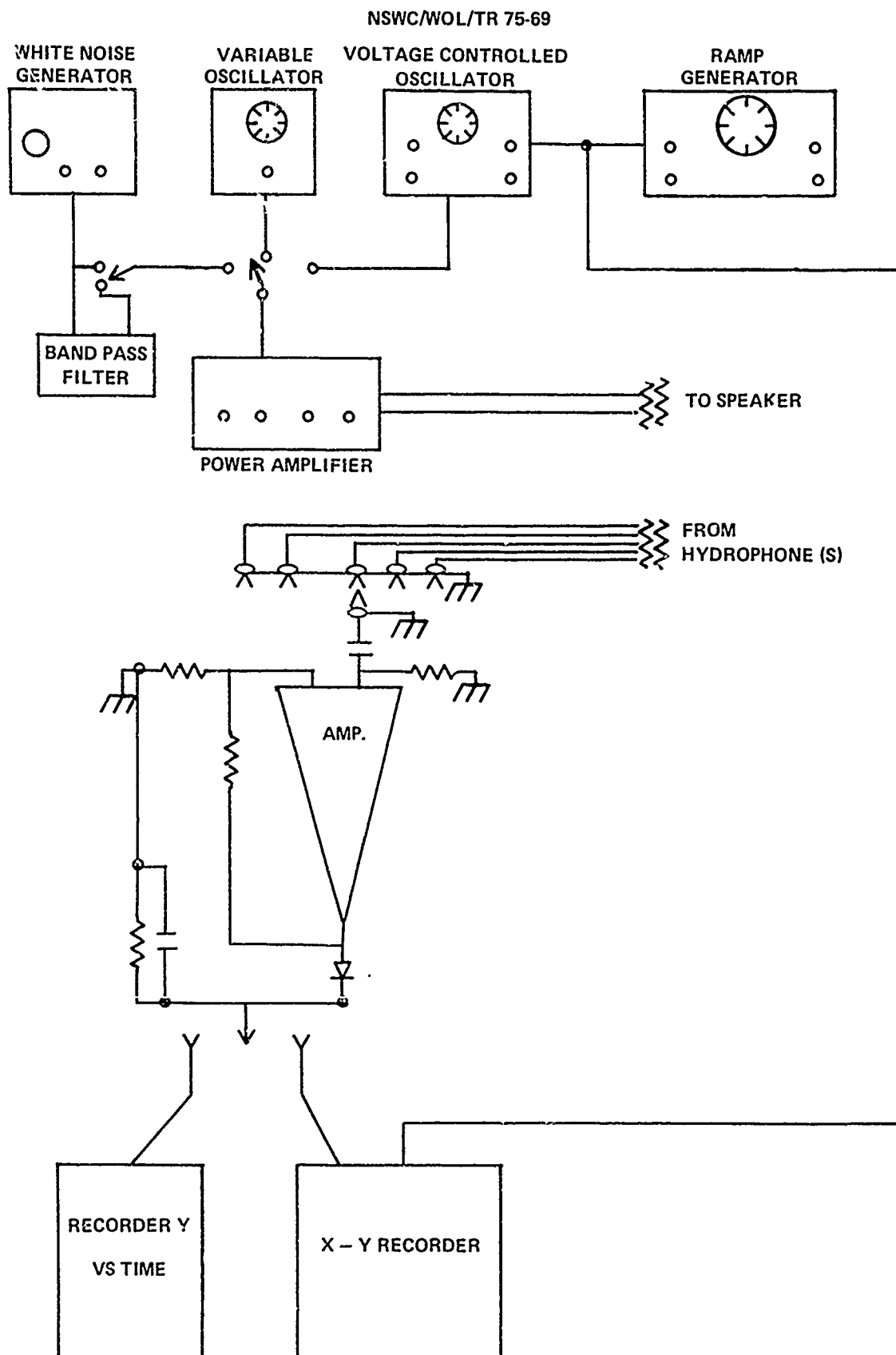


FIG. 9 THE ELECTRONIC DATA PROCESSING CIRCUITRY

the sound transmission at 1800 Hz. Such pulling could occur because a sizeable volume of average sound velocity below the pure water sound velocity could change the average sound speed in the tank and thus shift (downward) the frequencies of the naturally occurring modes. Since the initial attempts at damping were only marginally successful, the problem of separating possible mode pulling effects from true sound enhancement by the bubbles was further investigated by repeating the previous experiments in the larger acoustic tank. It had a major mode at about 1000 Hz and, of course, a complex spectrum above that frequency, but in this case it was relatively easy to separate the effects of the bubbles from the "background" spectra. It was found that in the large tank the bubbles increased the sound transmission in the 1500-2500 Hz band as well as at some higher frequencies.

The bubbles were generated by the random mixing of air and (soapy) water by the impeller of a centrifugal pump. As illustrated in Figure 8, water was siphoned at a constant rate (depending upon the relative water surface heights in the tank and in the pump container) into the pump container. The intake hole of the pump was only partially underwater, so its efficiency depended upon the height of the water in the container. It mixed air and water and forced them through the connecting hose back into the tank through the bubble port. The system was self-regulating as to flow rate and therefore maintained a nearly constant flow of bubbly water into the tank. Occasionally the flow rate would oscillate, but it never oscillated by more than an estimated 20% or so. Measurements indicated that the typical flow rate was about $18 \text{ cm}^3/\text{sec}$ corresponding to a flow velocity in the hose (0.28 cm^2 cross-sectional area) of about 64 cm/sec when the difference in water level heights between the tank and the pump container was about 50 cm.

During the latter part of the experimentation the fractional bubble volume of the water being pumped to the tank was measured by a volumetric measurement using a vertical glass tube about a meter long. By closing the hose at point A (Figure 8) and simultaneously opening the hose at point B the bubbly water was diverted into the vertical tube. When the level had risen about a meter above point A (several seconds after opening A) the hose was again clamped at B and the level was immediately marked. As the bubbles rose to the top of the tube and departed from the water, the height of the fluid dropped and, to a reasonably good degree of accuracy, the fractional bubble volume was read as the maximum decrease in fluid height over the initial fluid height. Fractional volumes greater than 0.002 were read in this way. To read lower bubble densities a more complex system would have been required, but it was felt that it was not worthwhile to devise such a system because (1) the acoustic effects of the bubbles were barely measureable with the apparatus used for β much below 0.002, and (2) for such small values of β a large proportion of the bubbles were large ($\sim 1 \text{ mm}$ diameter). When sufficient soap was added to make $\beta > 0.005$ the fractional volume was about evenly divided between large bubbles and small ($\sim 0.1 \text{ mm}$) bubbles. For $\beta > 0.01$ many of the bubbles were very small ($\sim 0.01 \text{ mm}$) and the largest were only about 0.1 mm in diameter.

The ejection angle of the bubble port was adjustable so that the bubble flow could enter into the tank vertically, horizontally, or at any angle in between. The bubble distribution in the tank always took on a conical shape, narrow at the bubble port and widening as the bubbles rose to the top. When the bubbles were ejected horizontally or nearly horizontally into the tank, the bubbly region was shaped like a cornucopia, as in Figure 8. The bubble density distribution perpendicular to the vertical or upward curved axis of the bubble column was such that the greatest density occurred at the axis of the cone and the density diminished in the outward direction from the axis until at a distance from the cone axis that depended upon how far the bubbles had travelled from the bubble port, the bubble density fell to zero. Thus the bubbly region did not have a well defined boundary.

There were two continuous problems with the experiments. Both of these problems were consequences of the existence of resonance modes of the water tanks. The first problem had to do with uncontrolled changes in the tank acoustic mode structure that were independent of the presence or absence of bubbles. Generally the main modes would remain at fixed frequencies but they would change in amplitude in an uncontrollable fashion despite the general stability of the mechanical apparatus and the maintenance of a constant water level in the tank. The second problem arose from the tendency of very tiny bubbles to dissolve or remain in suspension in the water and thus change its characteristics slightly. In particular, it would take the acoustic transmission spectrum several hours after bubbles were turned off to return to the shape it had before the bubbles were turned on. Because of this "relaxation time", and because the experiment consisted of comparing the spectrum in the presence of bubbles with the spectrum in the complete absence of bubbles, experiments could not be done in rapid succession. Figures 10A, 10B, 10C, and 10D illustrate the spectrum relaxation effect in the small tank. The mode spectrum before the addition of bubbles is illustrated in Figure 10A. The smooth dark line drawn through the spectrum is repeated in the following three figures to facilitate the comparison of the successive spectra with the pre-bubble spectrum. Figure 10B illustrates the spectrum in the presence of bubbles. The average β of the bubbly water flowing into the tank was about 0.007. It is apparent that the bubbles increased the overall sound power transmitted to the hydrophone in the frequency range from about 2000 to 3000 Hz. The bubbles may also have shifted the mode that was at about 2300 Hz to about 2050 Hz and they appear to have suppressed the amplitude of the mode at about 3200 Hz. Figure 10C illustrates the spectrum four hours after the bubbles were turned off. The spectrum has nearly recovered to its original shape, but the relative amplitudes of the two major is not yet correct. Figure 10D shows that after about eight hours the spectrum has essentially returned to its original shape. Because of this rather long spectrum relaxation time, when the greatest accuracy was desired, only one or two bubble experiments were done in one day.

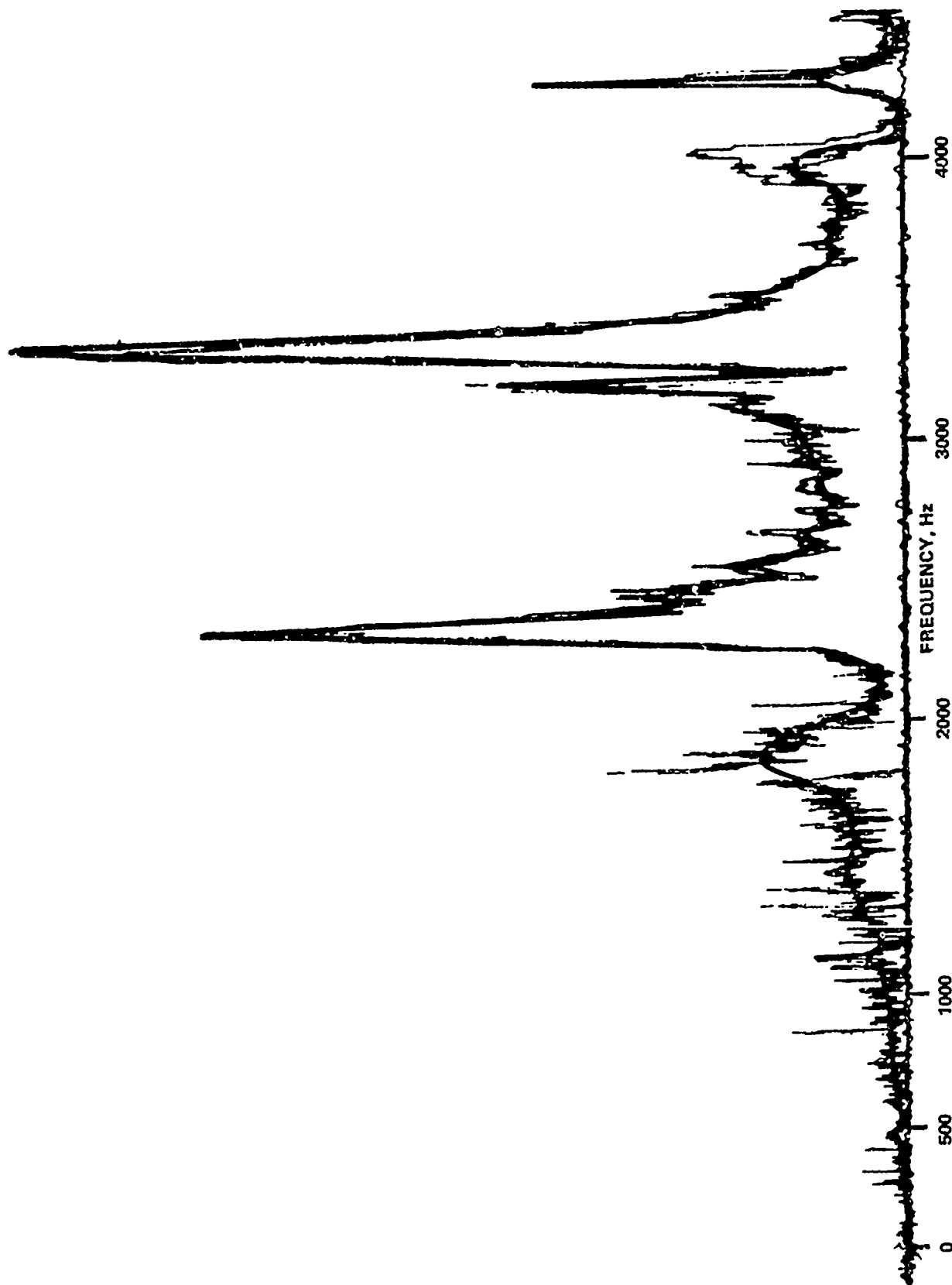


FIG. 10A ACOUSTIC SPECTRUM BEFORE BUBBLES

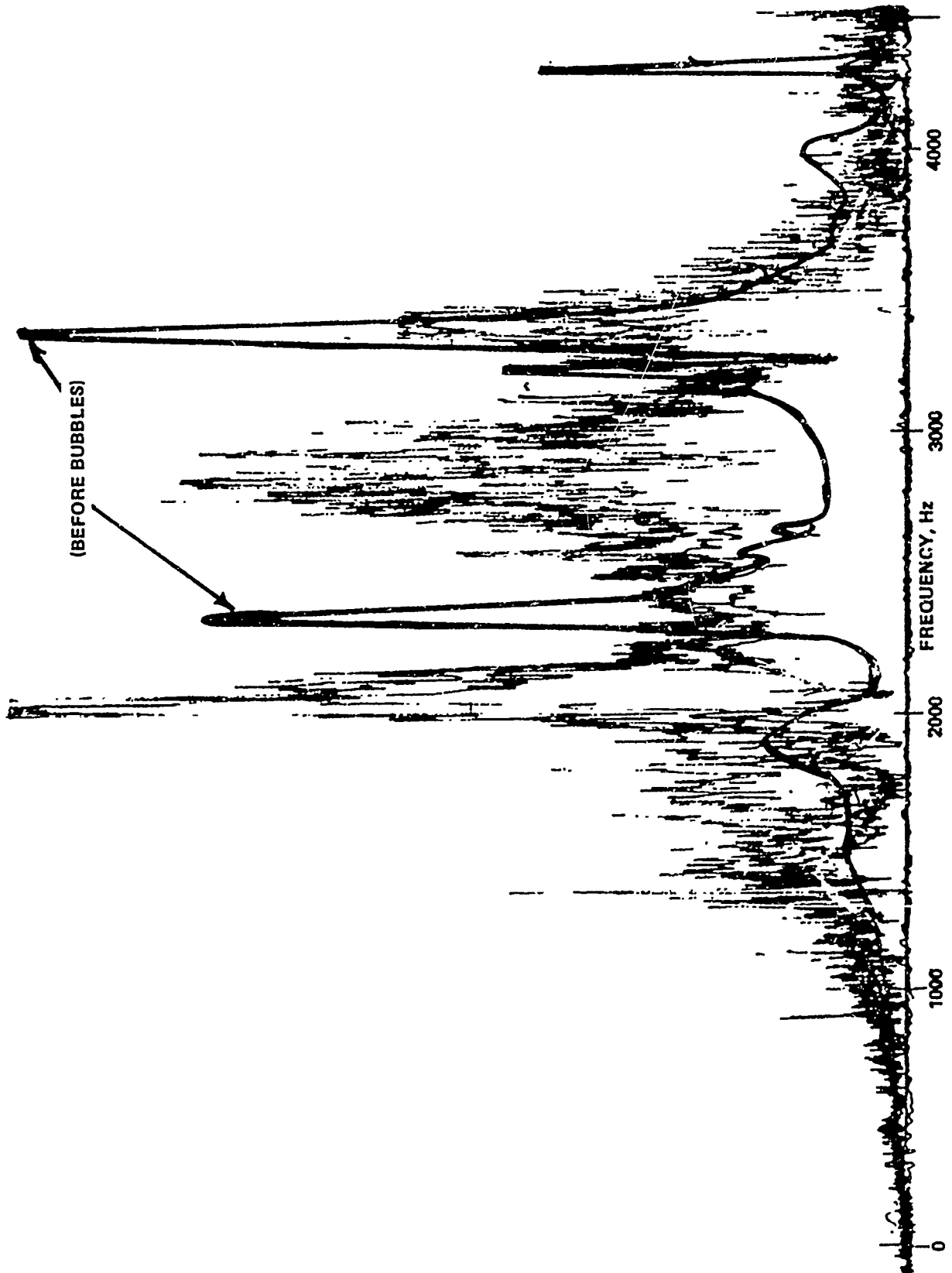


FIG. 108 ACOUSTIC SPECTRUM WITH BUBBLES

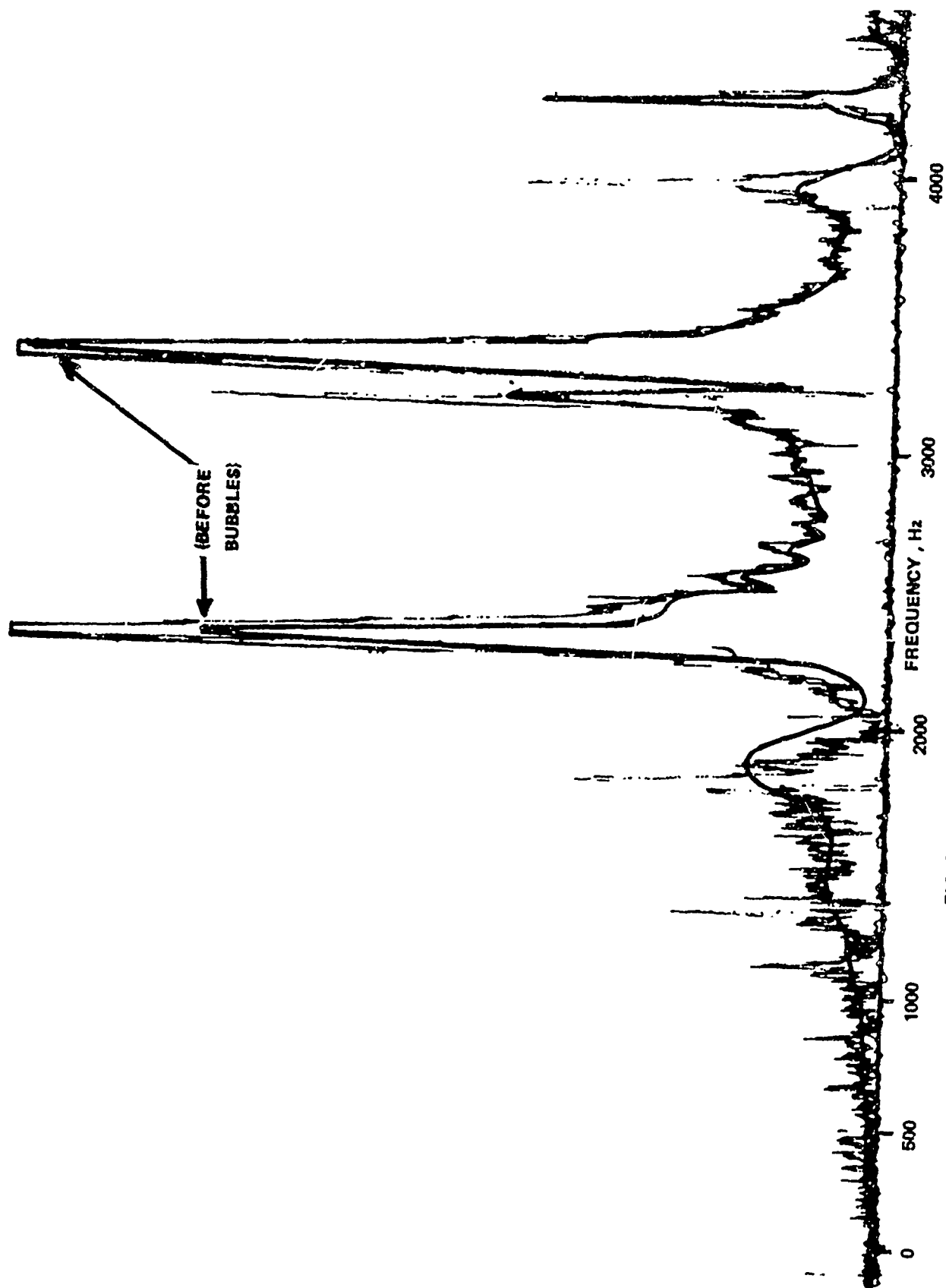
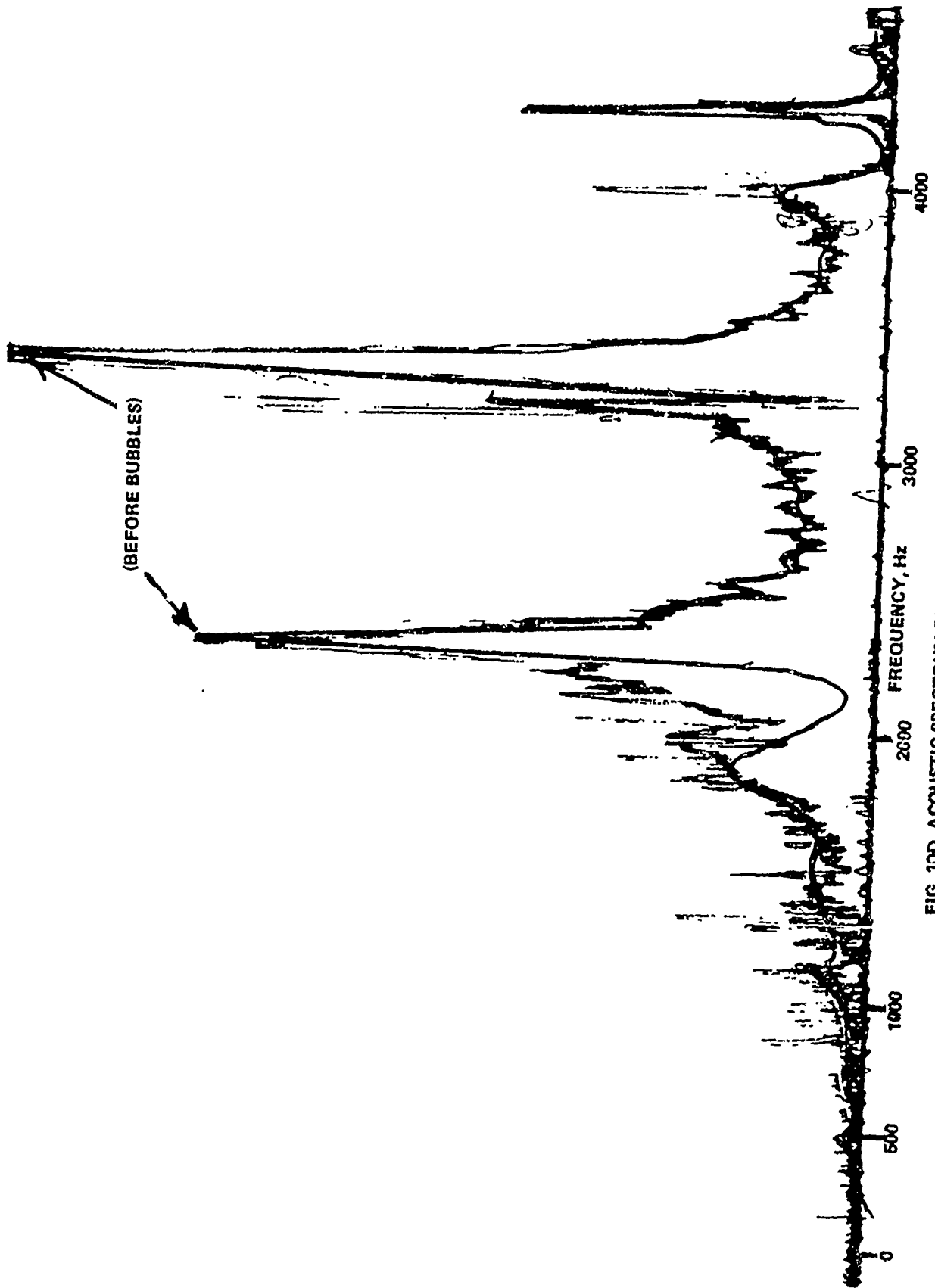


FIG. 10C ACOUSTIC SPECTRUM FOUR HOURS AFTER BUBBLES



One problem which was expected but turned out to be unimportant was the noise of the bubbles themselves. At no time was the small bubble noise sufficient to affect the spectrum, and typically, it was so small as to be barely detected at all. Figure 10B shows the noise level produced by the bubbles (the baseline of the figure). The noise in this baseline is comparable with the noise in the baselines of Figures 10A, 10C, and 10D.

IV. EXPERIMENTAL RESULTS

(a) The Transmission Of Sound Into The Large Tank

The effect of a column of bubbles on the transmission of sound into a large tank is most clearly illustrated in Figures 11 through 15. Figure 11 shows the experimental arrangement in the large acoustic tank. The origin of the coordinate system is at the bubble pipe exit port. For these figures the water height, z_w , was 56 cm, the speaker height, z_s , was 72 cm, and the speaker was "downstream" from the bubble port by $x_s = 30$ cm. The bubbles exited horizontally from the bubble port and "fell upward" until they reached the surface just under the speaker, as shown in Figure 11. The fractional volume at the bubble port was estimated to be about 0.01. The dependence of the acoustical spectra on the position of the hydrophone was determined in the absence and then in the presence of bubbles. The hydrophone positions are denoted by the values of x , y , and z on the figures. The hydrophone, an LC-32, was about 4 cm long by 1 cm in diameter.

Figure 12 demonstrates the frequency and spatial variations in the acoustic (pressure) transmission that are caused by the bubbles. In Figure 12A (and subsequent figures) the listed x , y , z coordinate positions are those of the hydrophone. A thick dark line has been drawn through the spectrum that was recorded when the bubbles were present. The essential features of Figure 12A are (1) the naturally occurring major acoustic mode of the box at roughly 1000 Hz, (2) the complicated mode structure at the higher frequencies, and (3) the increase in signal transmission when bubbles are present over when they are not present. This increase is especially noticeable in the frequency range 1800-2200 Hz. The spectrum without bubbles is a single trace on Figure 12A, but the spectrum with bubbles is shown by two successive traces. The time necessary to trace out the spectrum was on the order of several minutes, so it is apparent that the spectrum with bubbles is quite reproducible. For Figure 12A the hydrophone was about 5 cm downstream from the bubble port. Since it was mounted with its axis vertical, it intersected the bubble stream as the stream "fell upward" along a parabolic path toward the top. For Figure 12B the hydrophone was still about 5 cm downstream, but it was also about 4 cm up, so that it was no longer within the bubble stream. Although the frequency spectrum without bubbles did not change significantly, the amplitude of spectrum with bubbles decreased in the sound pressure transmission in the range 1800-2200 Hz, as well as

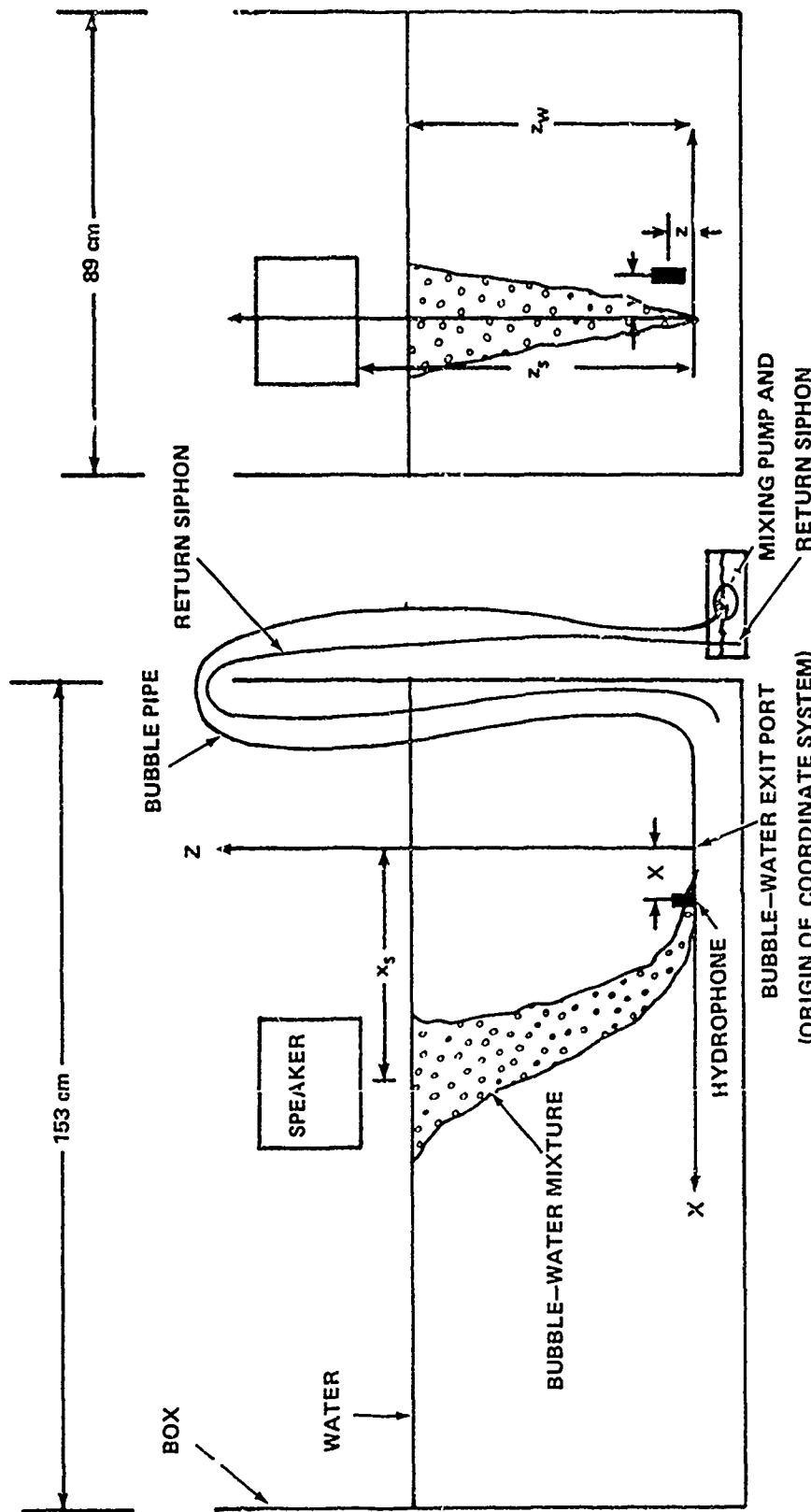


FIG. 11 ACOUSTIC TEST APPARATUS

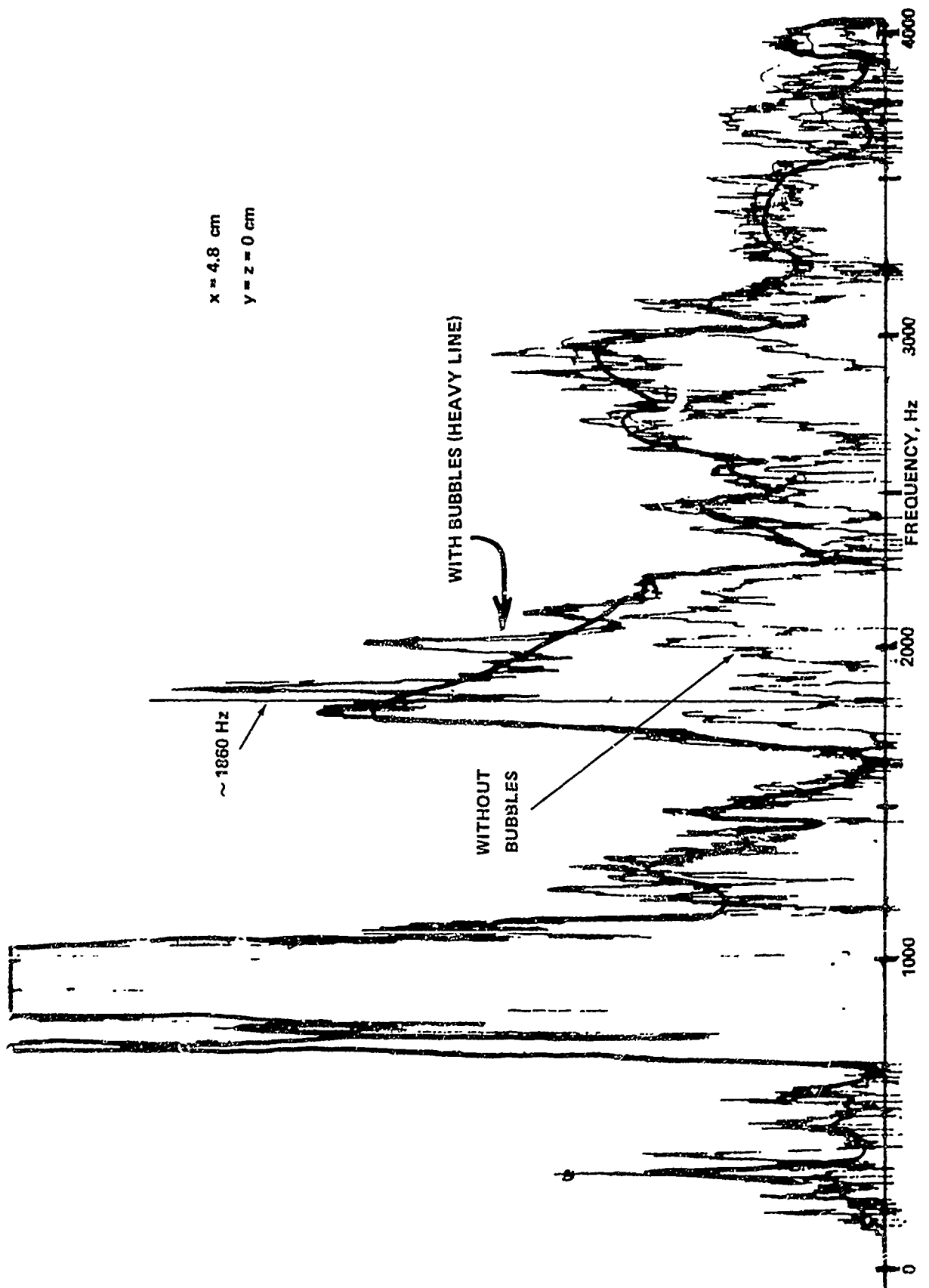
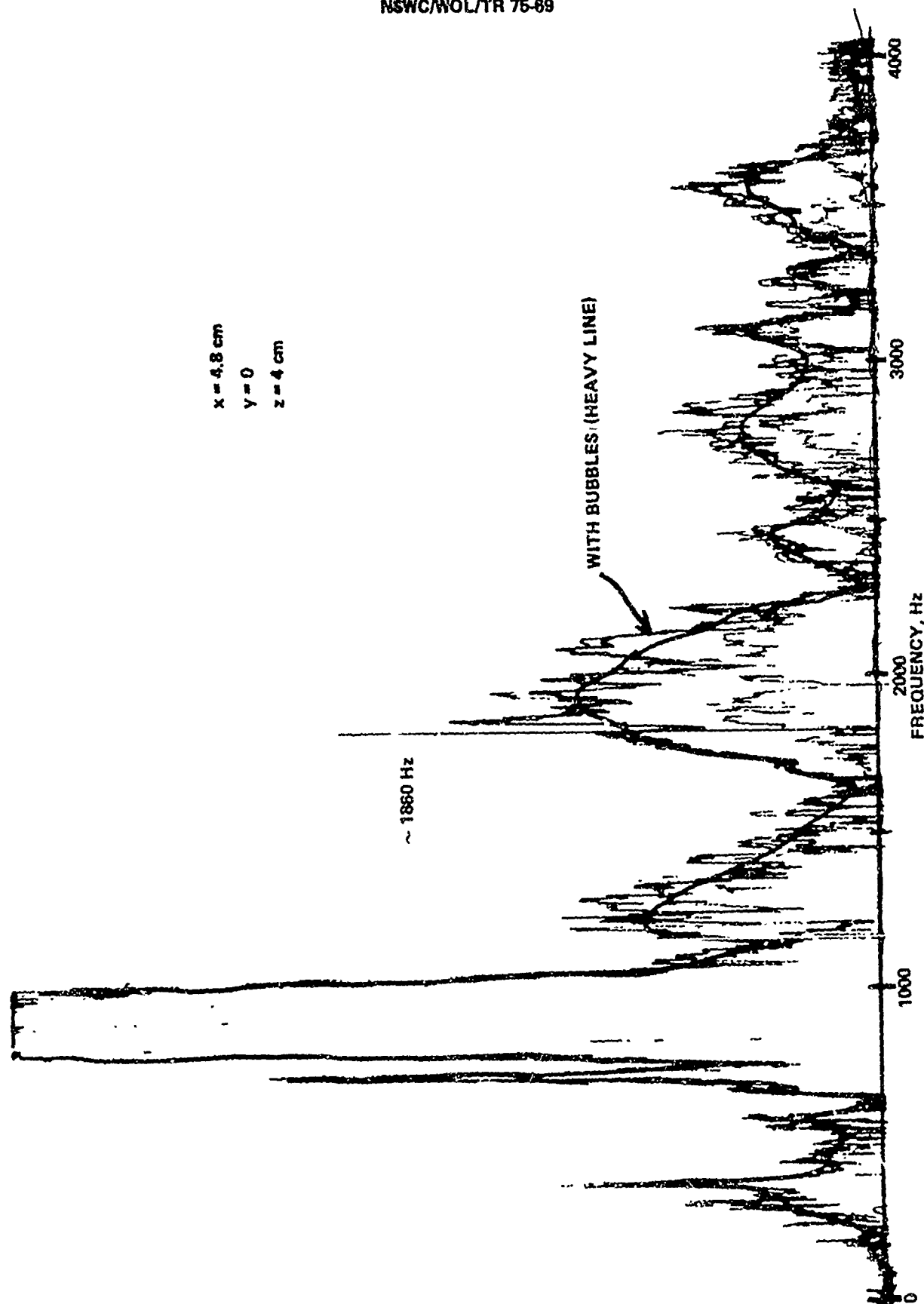


FIG. 12A SPECTRAL DEPENDENCE OF ACOUSTIC TRANSMISSION



x = 4.8 cm
y = 0
z = 4 cm

FIG. 12B SPECTRAL DEPENDENCE OF ACOUSTIC TRANSMISSION

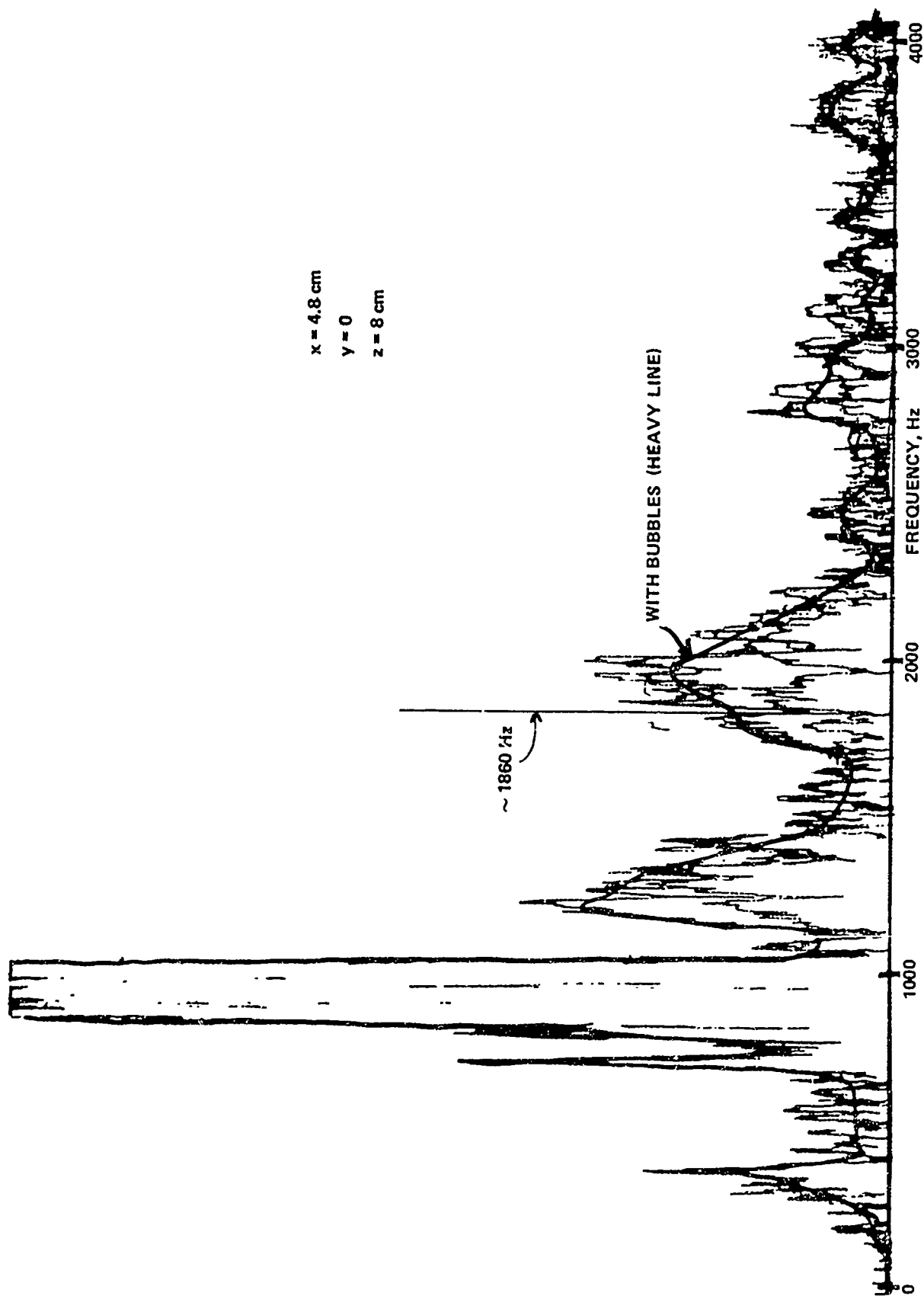


FIG. 12C SPECTRAL DEPENDENCE OF ACOUSTIC TRANSMISSION

in higher frequency bands. The lowered acoustic transmission outside the bubble stream is even more noticeable in Figure 12C. The hydrophone was then 8 cm from the bubble stream and the SGF in the 1800-2200 Hz band was only about 50% of what it was in the same band in Figure 12A. The decrease in sound transmission as the hydrophone was moved away from the bubble stream occurred despite the fact that raising the hydrophone moved it closer to the speaker. (Note: the $(\cos \theta)/r$ factor in the expected radiation pattern of the acoustic pressure in water below the speaker^{15,20} predicts, in the absence of tank resonances, an increase in sound pressure of about 8% when the hydrophone is raised 8 cm.)

Figures 13A, B, C, and D illustrate the time development of the underwater acoustic pressure when the bubble stream is turned on. Only a single frequency was monitored, $f = 1860$ Hz. This frequency was noted as a vertical line in Figures 12A, B, and C. On Figure 13A we see that about 10 seconds after the bubbles were turned on the sound pressure began to increase noticeably (it took about 10 seconds for the bubble column to become established in the large tank). From then on the increase in sound pressure was slow but continual, suggesting a gradual increase in the average bubble fraction, β . The pressure increase stabilized after about seven minutes. (Note: the random fluctuations in the sound pressure with bubbles, were caused by uncontrolled changes in the bubble flow rate and probably also uncontrolled changes in the bubble column distribution in the tank. Random fluctuations when the bubbles were not present were caused by extraneous noises picked up by the hydrophone.) After nine minutes the bubbles were turned off (Figure 13B) and the sound pressure decreased to essentially its original value within about 30 seconds. When they were again turned on the sound pressure increased and stabilized within about a minute at a value slightly lower than the average peak pressure that had been recorded seven minutes after the bubbles had been first turned on. The average signal gain factor (SGF) during the period six to nine minutes after the bubbles were first turned on was about 18-19, and the SGF after the bubbles had been turned on the second time was about 17 (see Figure 13B).

Figures 13C and D illustrate the effect of having the hydrophone outside the bubble stream. In Figure 13C the hydrophone was about 4 cm above the bubble stream and the SGF was only about 11. When the hydrophone was 8 cm above the bubble stream the SGF was about 8 (see Figure 13D). Thus, in moving the hydrophone 8 cm away from the bubble stream the signal gain factor dropped by about 50%, even though the decreased distance to the sound source would have increased the sound pressure by about 8% in the absence of acoustic

²⁰R.J. Urick, J. Acous. Soc. Amer. 52, 993 (1972)

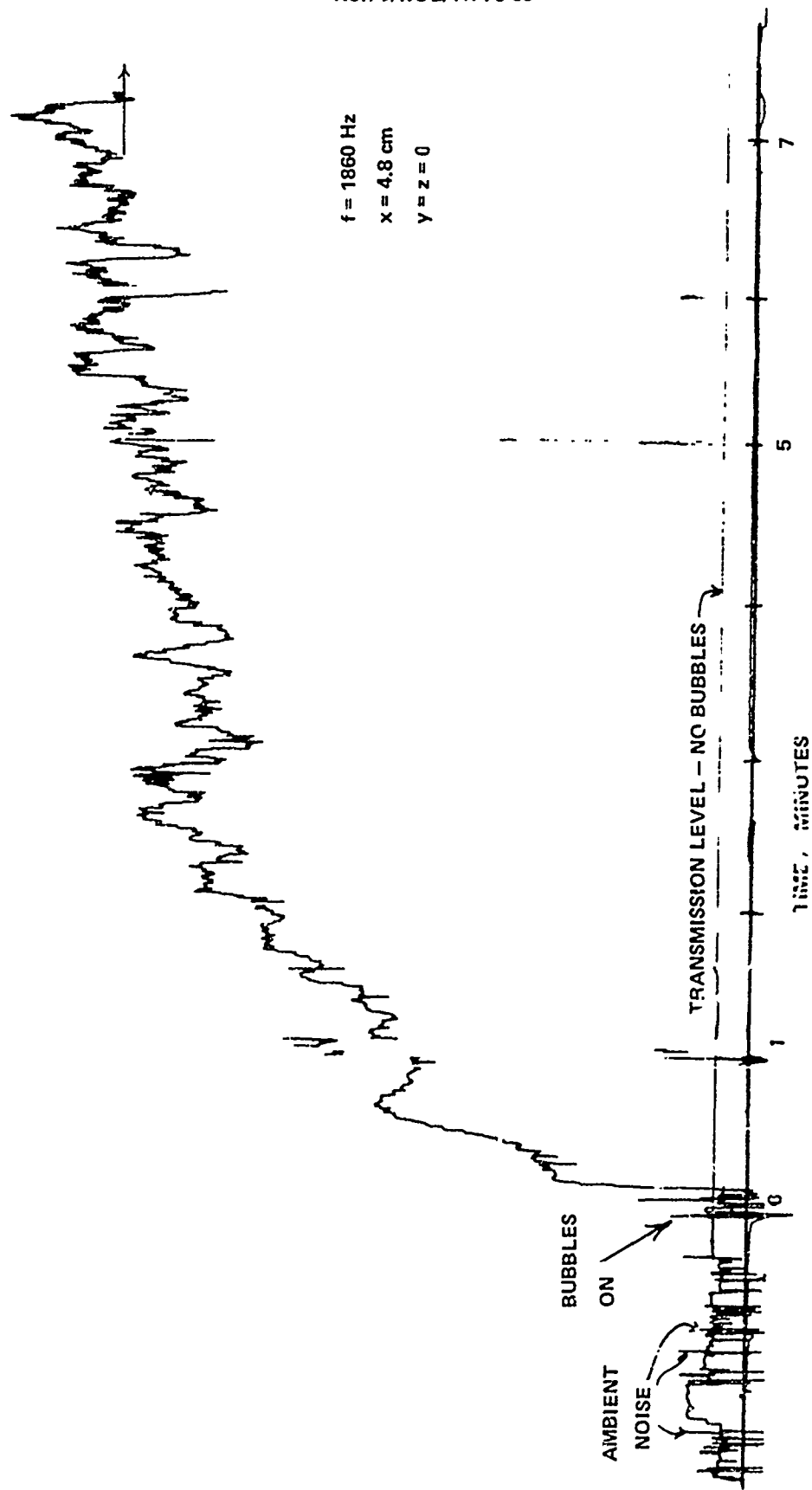


FIG. 13A TIME DEPENDENCE OF ACOUSTIC TRANSMISSION

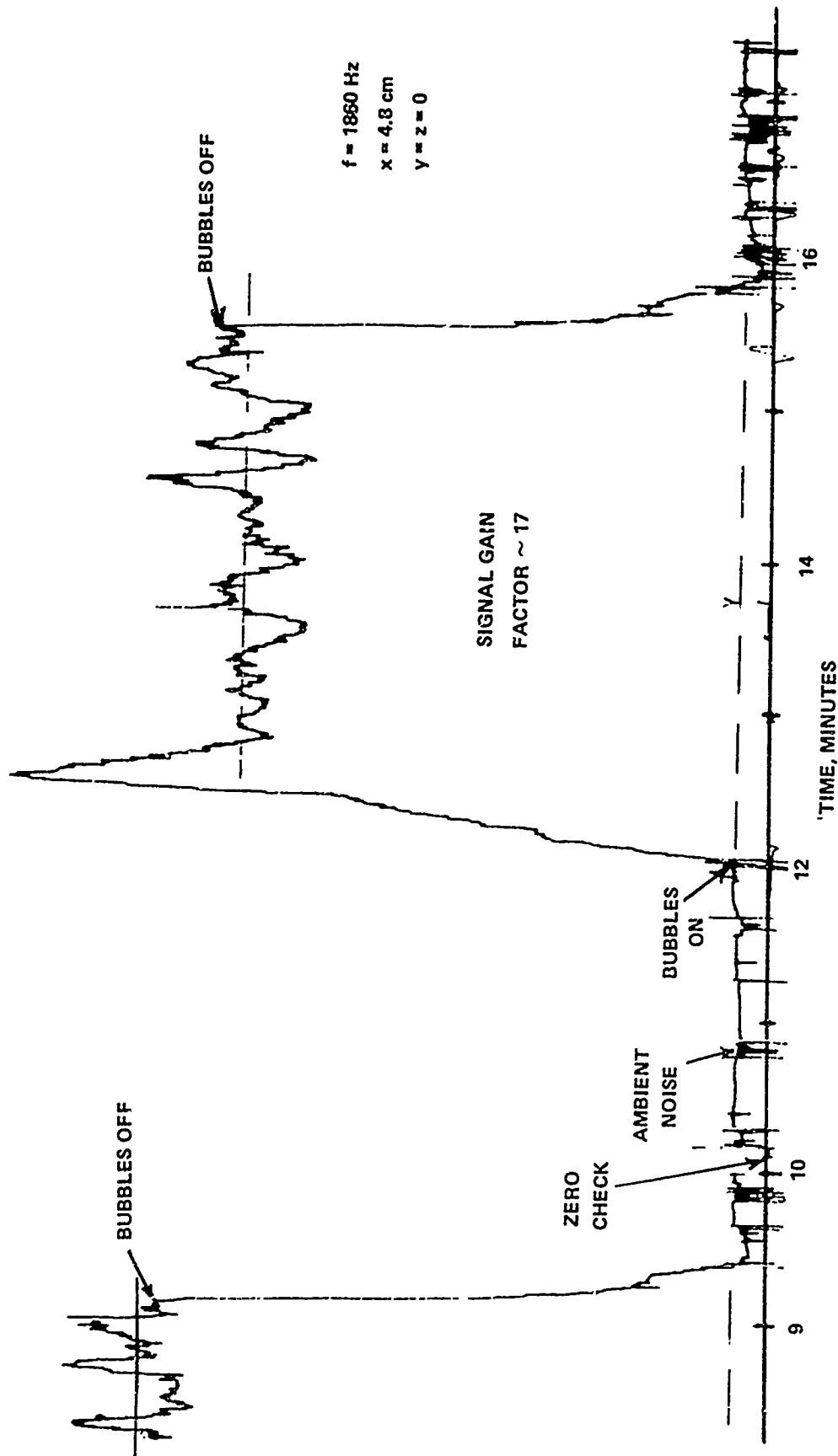


FIG. 13B TIME DEPENDENCE OF ACOUSTIC TRANSMISSION

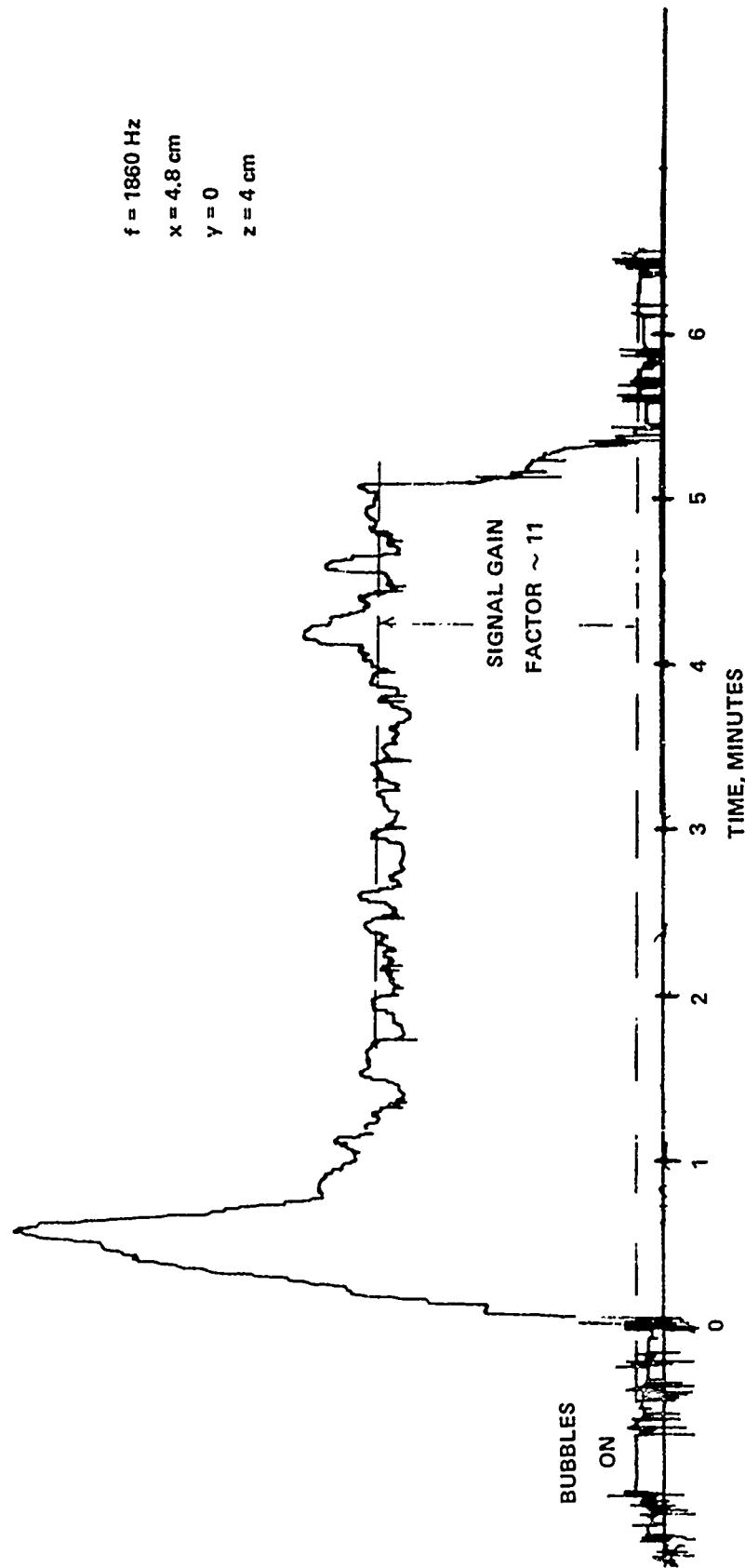


FIG. 13C TIME DEPENDANCE OF ACOUSTIC TRANSMISSION

tank modes. The observed decrease in amplitude as the hydrophone is moved out of the bubble stream is suggestive of a degree of sound "trapping" by the bubbly region of the water.

In order to average over the box modes and thus to determine the effect of the bubbles on the transmission of a band of frequencies, the noise generator and an electronic filter (see Figure 9) were used to produce a band limited noise spectrum that was about one-half an octave wide at the 3 db points. With the center frequency set at 1900 Hz the time dependence of the acoustic transmission shown in Figure 14 was obtained. The signal gain factor averaged over the band (roughly 1500-2500 Hz) was only 1.6, which is to be compared with 17 (Figure 13B) at the specific frequency 1860 Hz. However, this very low signal gain over a rather wide band is not completely unexpected because, as is shown in Figure 12A, outside the frequency band from about 1700 Hz to about 2300 Hz the signal gain is less than or comparable to 1.6 (except in the range 2700-3000 Hz, which was outside the range of the filter).

Despite care in the construction and maintenance of the physical arrangements of the apparatus for the various experiments that were performed, uncontrolled changes in magnitudes of the effects of the bubbles were noticed. Figures 15A and B illustrate such a change. They were made several days after Figures 13 and 14 with the same apparatus, but perhaps with a different bubble density (bubble density measurements were not being made at the time of these experiments). Figure 15A is an expanded spectrum version of Figure 12A. In Figure 15A the spectrum without the bubbles has been emphasized with the heavy dark line. Several spectra without and then with bubbles were made (each spectrum took several minutes to plot). The repeatability of the main features of the spectra are apparent, as is the signal increase in the frequency range 1700-2200 Hz when bubbles were present. A particular frequency, 1900 Hz, was then selected for a sound pressure vs. time experiment. The results of this experiment are shown in Figure 15B. Qualitatively Figure 15B is the same as Figure 13B where a signal gain factor of 17 was measured. The lower signal gain factor in Figure 15B may have been due to a decrease in bubble density, but if so it was an unexpected decrease since the bubble making apparatus was the same as for Figure 13B. Such unexpected changes occurred frequently during the experimentation and made it difficult to determine quantitative effects of the bubbles on the sound transmission.

(b) The Transmission Spectra In An Acoustically Damped Tank

As has been stated previously, the major experimental difficulty was the determination of a possible spectral "preference" of the bubble distribution for the enhancement of the transmitted acoustic pressure. Experiments in a large and in a small acoustic tank suggested that the bubble column increased the sound transmission in a band centered in the range 1800-2000 Hz regardless of the size of the tank. However, mode "pulling" could not be ruled out as a possibility in either

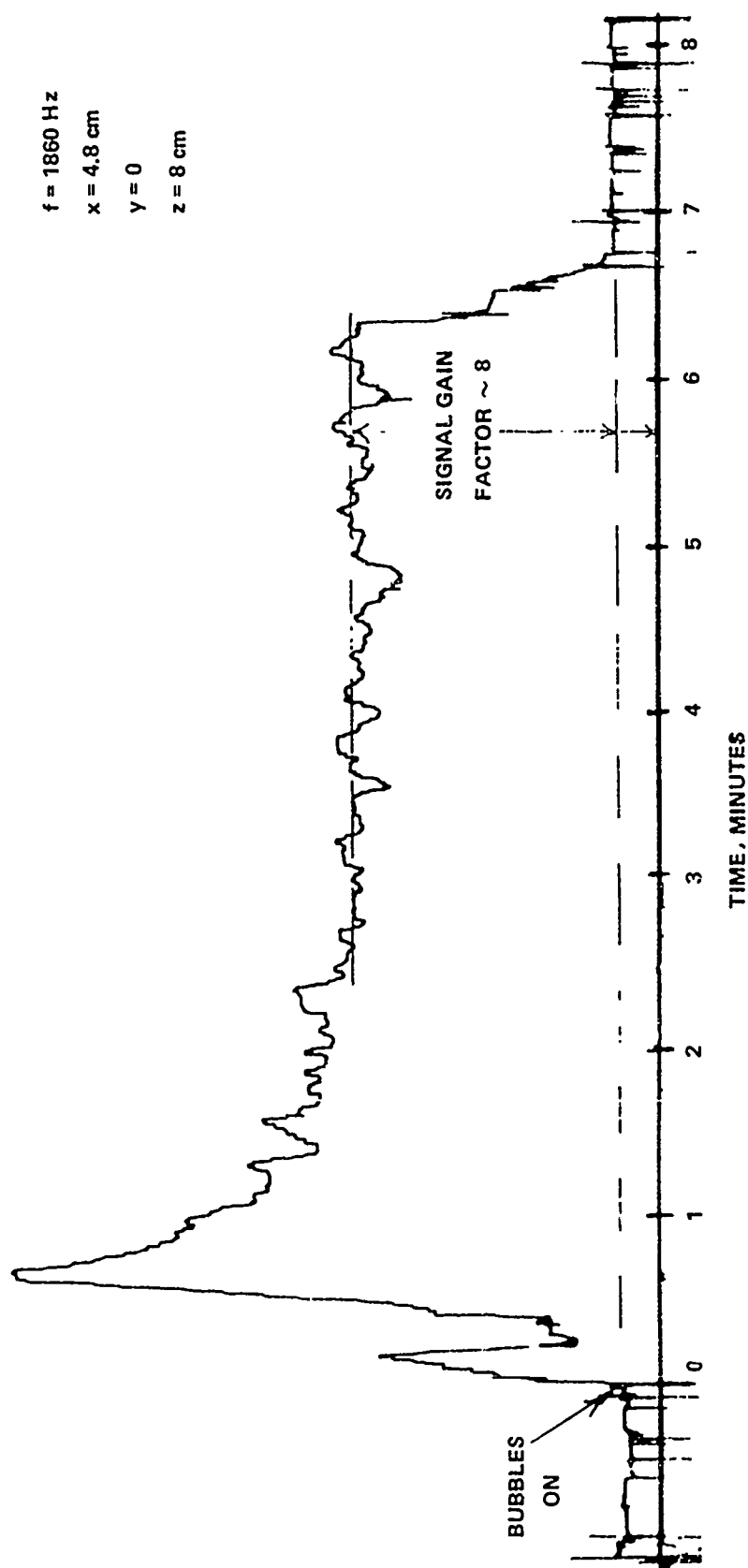


FIG. 13D TIME DEPENDANCE OF ACOUSTIC TRANSMISSION

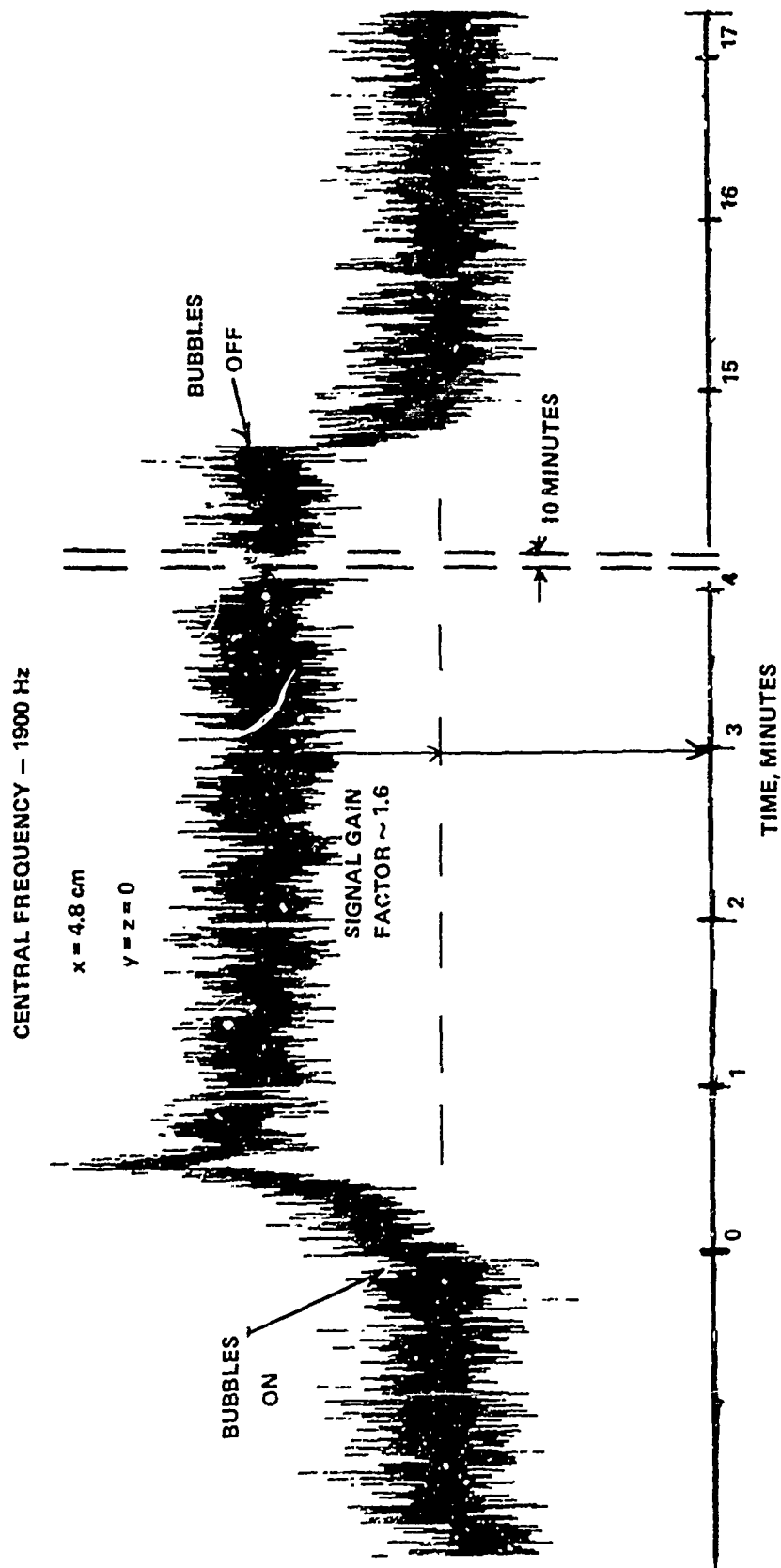


FIG. 14 THE ACOUSTIC TRANSMISSION OF A SPECTRAL BAND

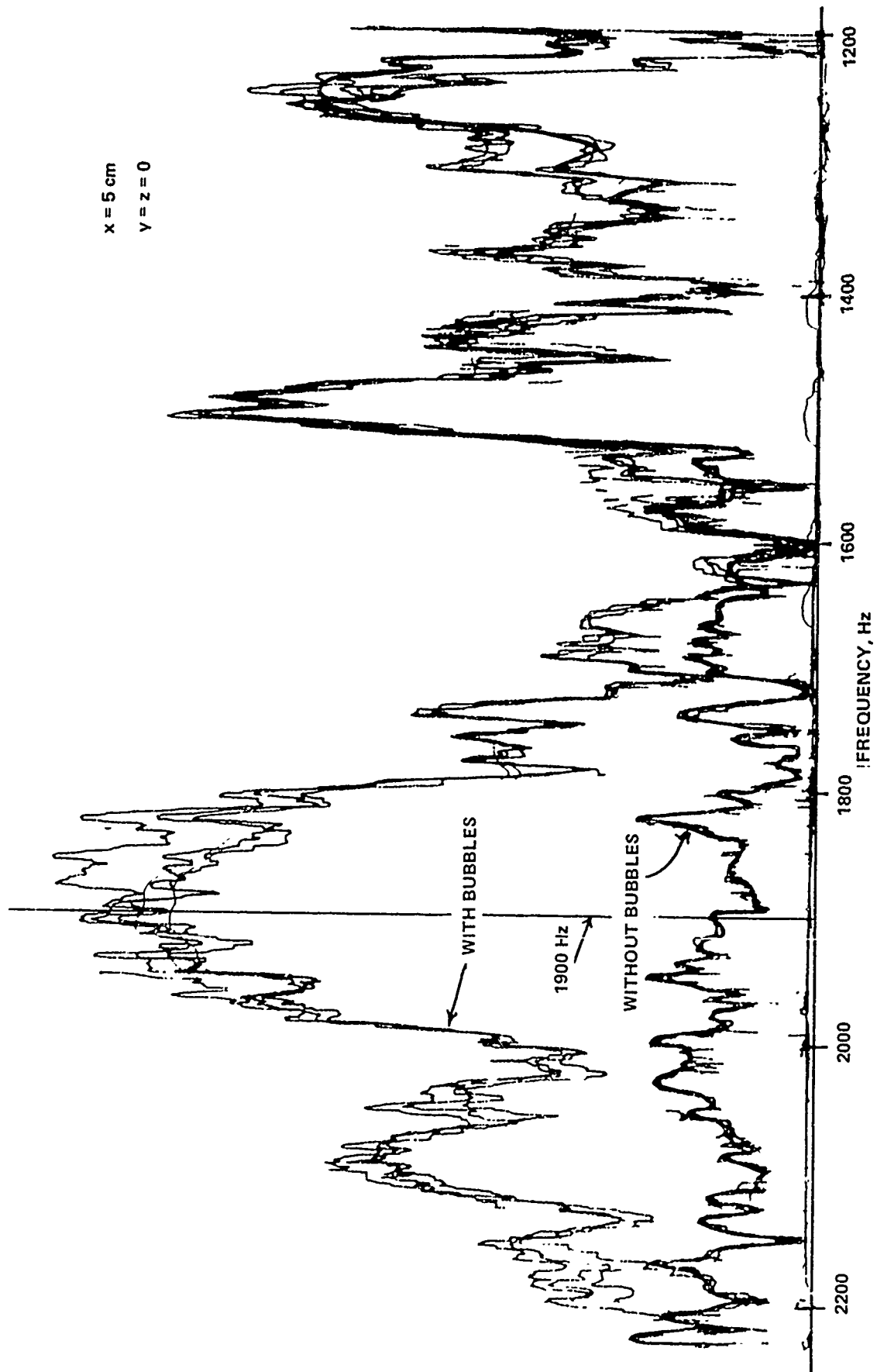


FIG. 15A SPECTRAL DEPENDENCE OF ACOUSTIC TRANSMISSION

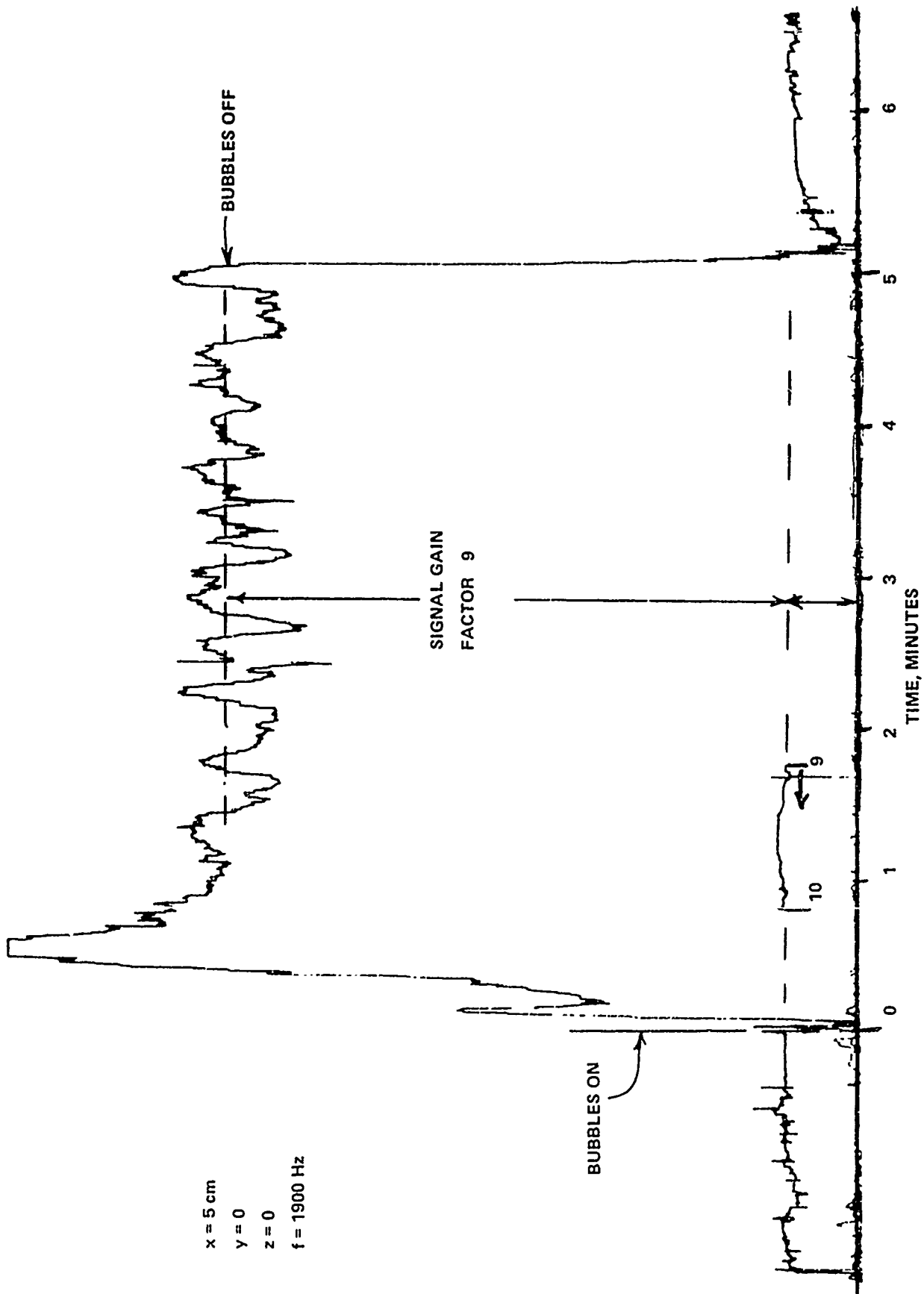


FIG. 15B TIME DEPENDENCE OF ACOUSTIC TRANSMISSION

$x = 5 \text{ cm}$
 $y = 0$
 $z = 0$
 $f = 1900 \text{ Hz}$

tank. Therefore, considerable effort was put into attempts to acoustically damp the small tank. Of course the main problem with acoustically damping a small tank is that the wavelengths at the frequencies of interest are large, much larger than the space available for typical acoustic damping devices such as rubber wedges, baffles, etc., and consequently the acoustic waves are only slightly scattered by the damping devices. Such damping devices would have an effect due to their (rather small) absorption, but the amount of this effect would hardly effect the basic mode structure of the tank. Nevertheless, experiments with several inches of dry foam rubber packing showed that when the material was "fresh", i.e., had just been placed along the walls inside the tank, it was quite effective in reducing the mode structure of the transmission spectrum. A comparison of Figure 16 with Figure 10A illustrates the effect of the foam. In Figure 16 the spectrum before the bubbles were added is emphasized with the heavy line. For Figure 16 the bubble column was nearly vertical from the bubble port up to the surface, the speaker was 3 cm above the top of the bubble column, and the hydrophone was 30 cm below the speaker at the level of the bubble port, but it was not within the bubble column. The lack of a complicated mode structure in Figure 16 as compared with Figure 10 is apparent. When bubbles were present in the damped tank the transmission of sound in the frequency band from about 1200 to about 3000 Hz was noticeably enhanced. Thus this experiment and others like it increased confidence in the previous suggestion that the bubble column itself exhibited a sort of broad-banded transmission resonance. Whether or not the resonance is an artifact of the bubble distribution (shape of the bubble column) is not at present known for certain. The apparatus used to produce the bubbles and to project them into the acoustic tank was not sufficiently flexible to allow for major changes in the shape of the bubble column.

The damping effect of the foam was only temporary. Gradually over a period of several weeks the spectrum in the absence of bubbles returned to its "pre-foam" complexity, as is illustrated by Figure 17, which was made about three weeks after Figure 16, and by Figure 10A, which was made about nine weeks after Figure 16. The spectra shown in these figures are not completely comparable since the arrangements of the bubble port, the hydrophone, and the speaker were different for all three figures. Nevertheless, the decrease with time of the degree of damping of the major tank modes is apparent.

(c) The Spatial Variation Of The Signal Fain Factor

It has already been pointed out that the SGF dropped when the hydrophone was raised above the bubble column, even though it was, at the same time, being moved closer to the speaker sound source (see Figures 12 and 13). Further experimentation with the apparatus of Figure 11 showed that the SGF decreased uniformly with distance in the horizontal "y" direction (see Figure 11) and even became less than one for y sufficiently far from the bubble stream. The results of two of these experiments are shown in Figure 18. In these experiments the

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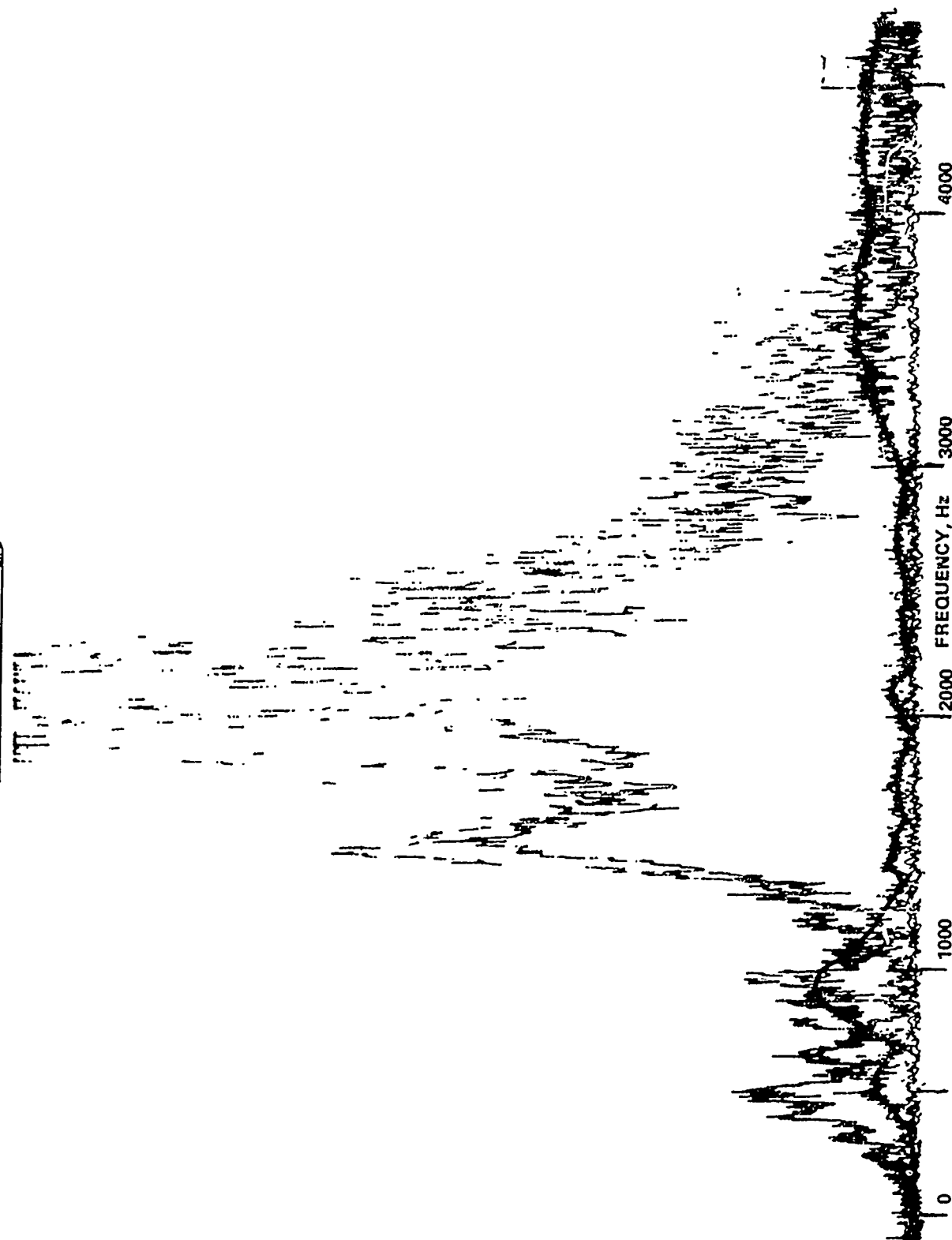


FIG. 16 TRANSMISSION SPECTRUM IN AN ACOUSTICALLY DAMPED TANK (THE EFFECT OF "FRESH" FOAM RUBBER DAMPING MATERIAL)

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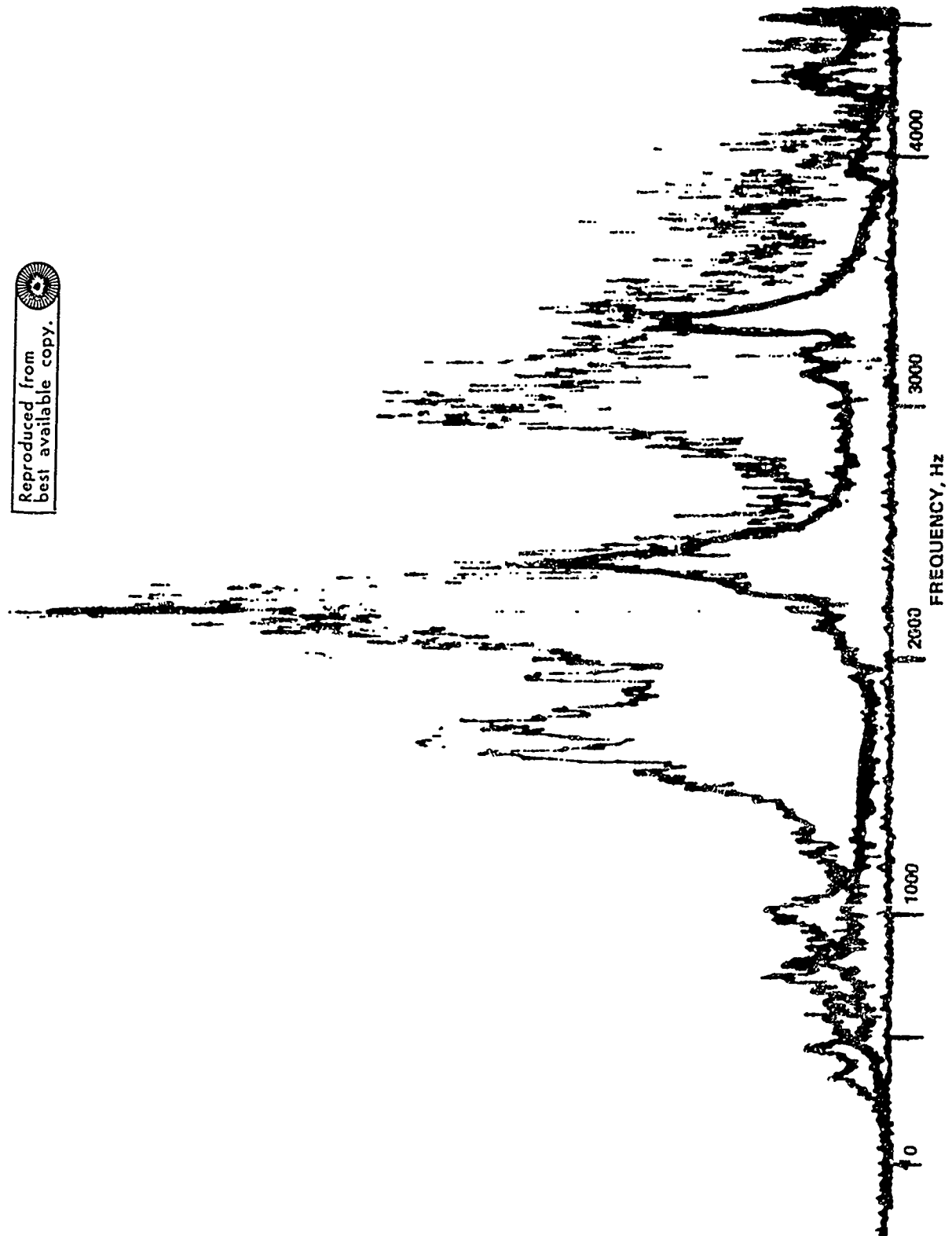


FIG. 17 TRANSMISSION SPECTRUM IN AN ACOUSTICALLY DAMPED TANK (THREE WEEKS AFTER FIG. 16)

speaker height, z_s , was 97 cm, the water height, z_w , was 58 cm, and the speaker x_s and y_s positions were 31 cm and 0 cm respectively. The hydrophone x and z positions were 1 cm and 0 cm respectively, and the y position varied from 2 cm on one side of the bubble port to about 20 cm on the other side of the bubble port. The frequency was constant at 2250 Hz. Each data point on the graph represents the ratio $SGF = p_{w,b}/p_w$. The experimental ratios represented by the triangles were obtained with a slightly lower β (as estimated from visual inspection of the bubble column) than were the ratios represented by the circles. A solid line has been drawn through the data points for illustration. The diameter of the bubble column at the x and z positions of the hydrophone was about 1 cm, so, according to Figure 18, the effects of the bubble column on the SGF extended to at least 40 times its own radius. Of particular interest is the region from 8 to 20 cm where the SGF was less than one. The bubbles actually decreased the sound pressure transmitted to that region. This suggests that the bubble column increased the sound intensity in the region from 0 to 8 cm at the expense of the intensity further away from the bubble column. This also suggests that in order to determine whether or not the bubble column actually increased the overall power transmitted into the water it would be necessary to measure the SGF at many places in the tank to determine the average power density in the tank with and without bubbles. Such measurements have not been done for this investigation.

(d) The Bubbly Region As A Leaky Waveguide

The preceding experiment suggested that the sound was being trapped by the bubbly region at the surface and then being guided, as in a "leaky" waveguide down to the apex of the conical bubbly region. Since the diameter of the bubbly region near its apex was much smaller than its diameter at the water surface, it was conjectured that the sound rays might also have been focused by the conical shape of the bubbly region. Measurements of the sound pressure just below the surface and near the apex of the bubbly region support this conjecture, as is illustrated in Figures 19-23. Figure 19 shows the experimental arrangement in the small tank. Spectra first without and then with bubbles were determined with the hydrophone at positions A and B. The spectra were recorded using several different values of β . Typical such spectra are shown in Figures 20-23. In each case the heavy line is the spectrum before the bubbles were injected into the tank. Several traces of the spectra with bubbles, each trace taking about one minute to complete, were made. The various traces are labeled on the figures to illustrate the time development of the overall spectra with bubbles. In general, it appeared that the spectral regions of large SGF moved toward lower frequencies as time went on. This effect will be discussed in following sections. Of interest here are the maximum SGF's at the frequencies indicated by the vertical dashed lines at position A compared with those at position B. Comparing Figures 20 and 22 (position A) with their respective (same β)

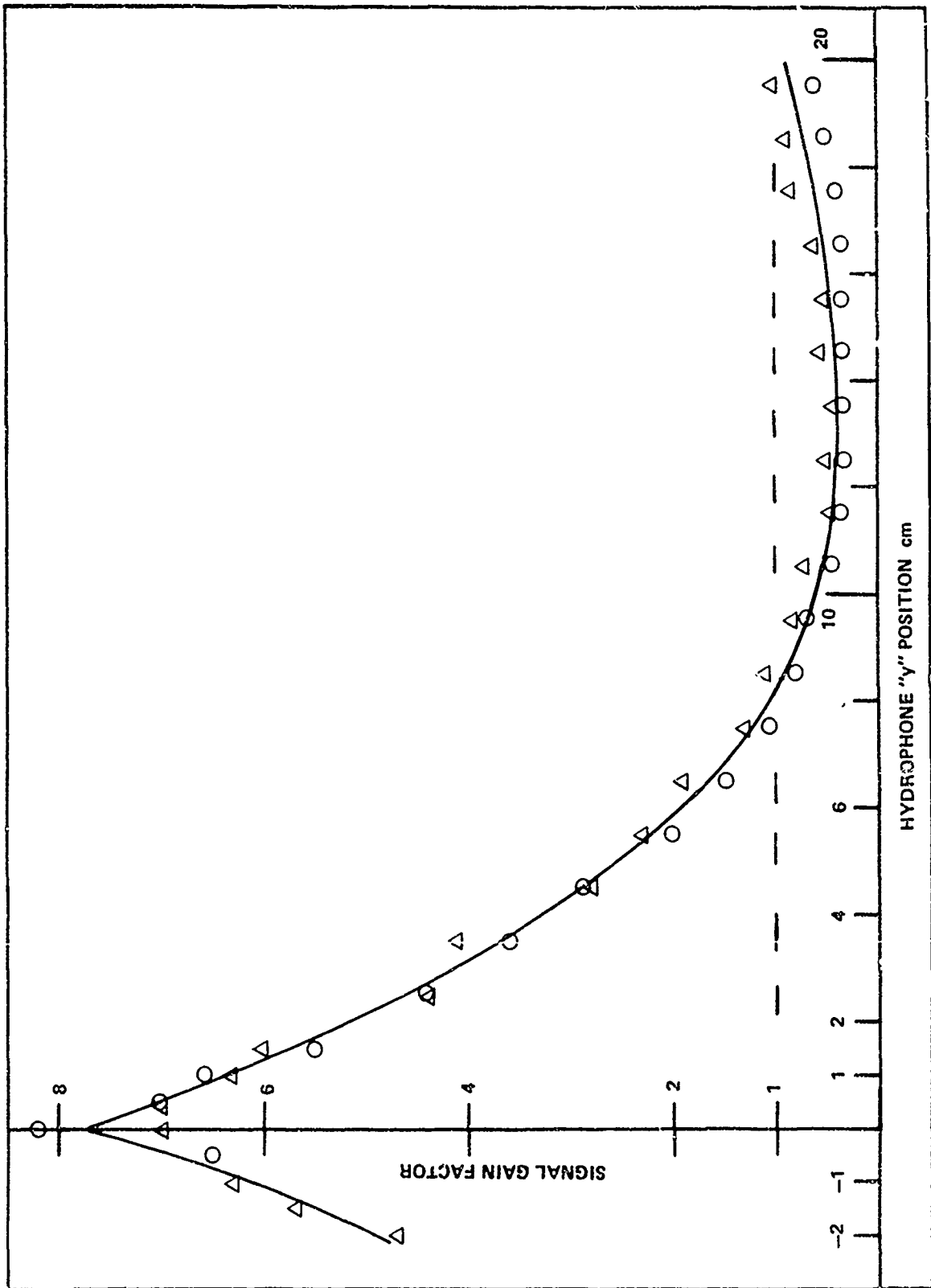


FIG. 18 SPATIAL VARIATION OF THE SIGNAL GAIN FACTOR

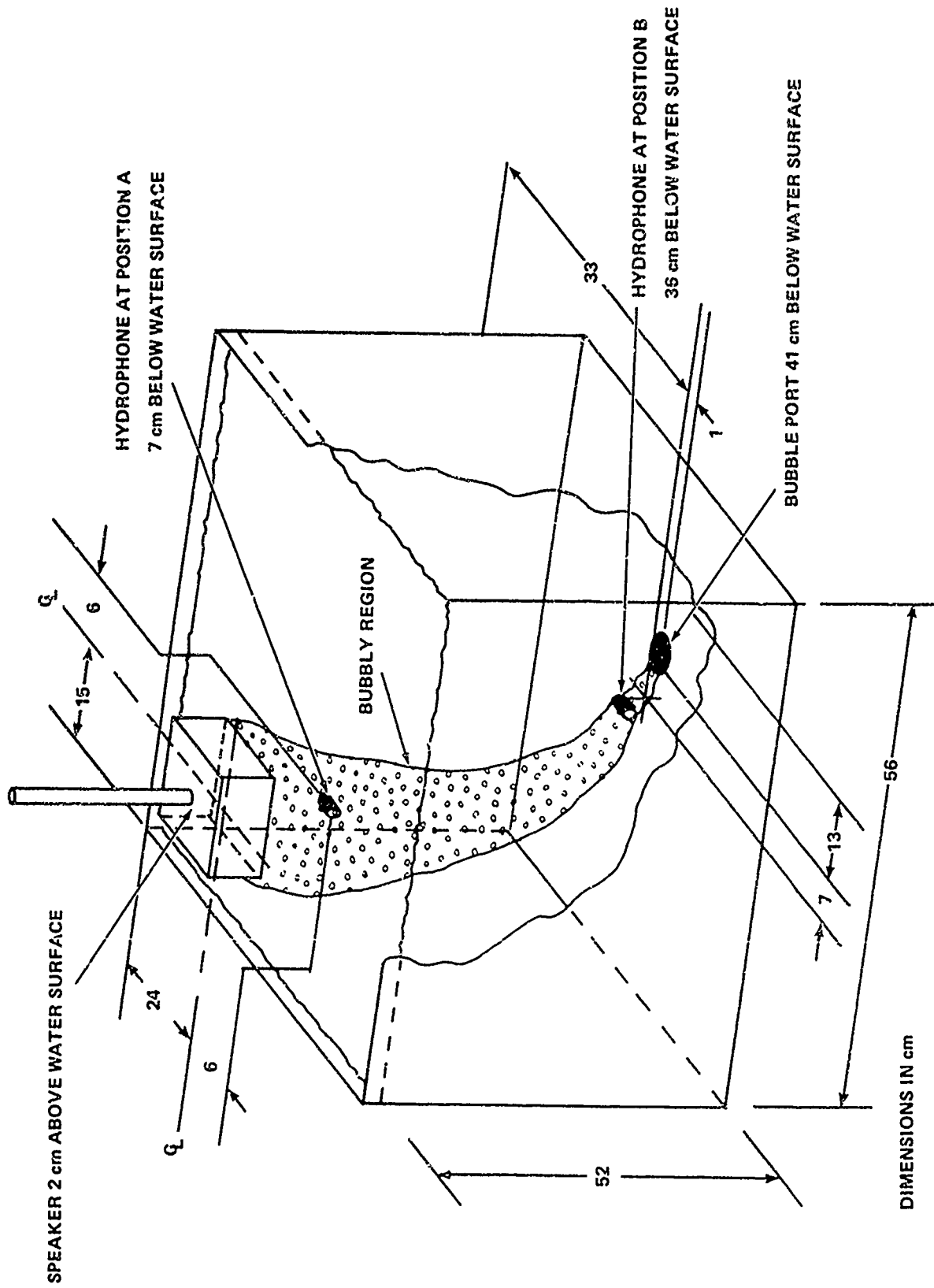


FIG. 19 THE SMALL ACOUSTIC TANK

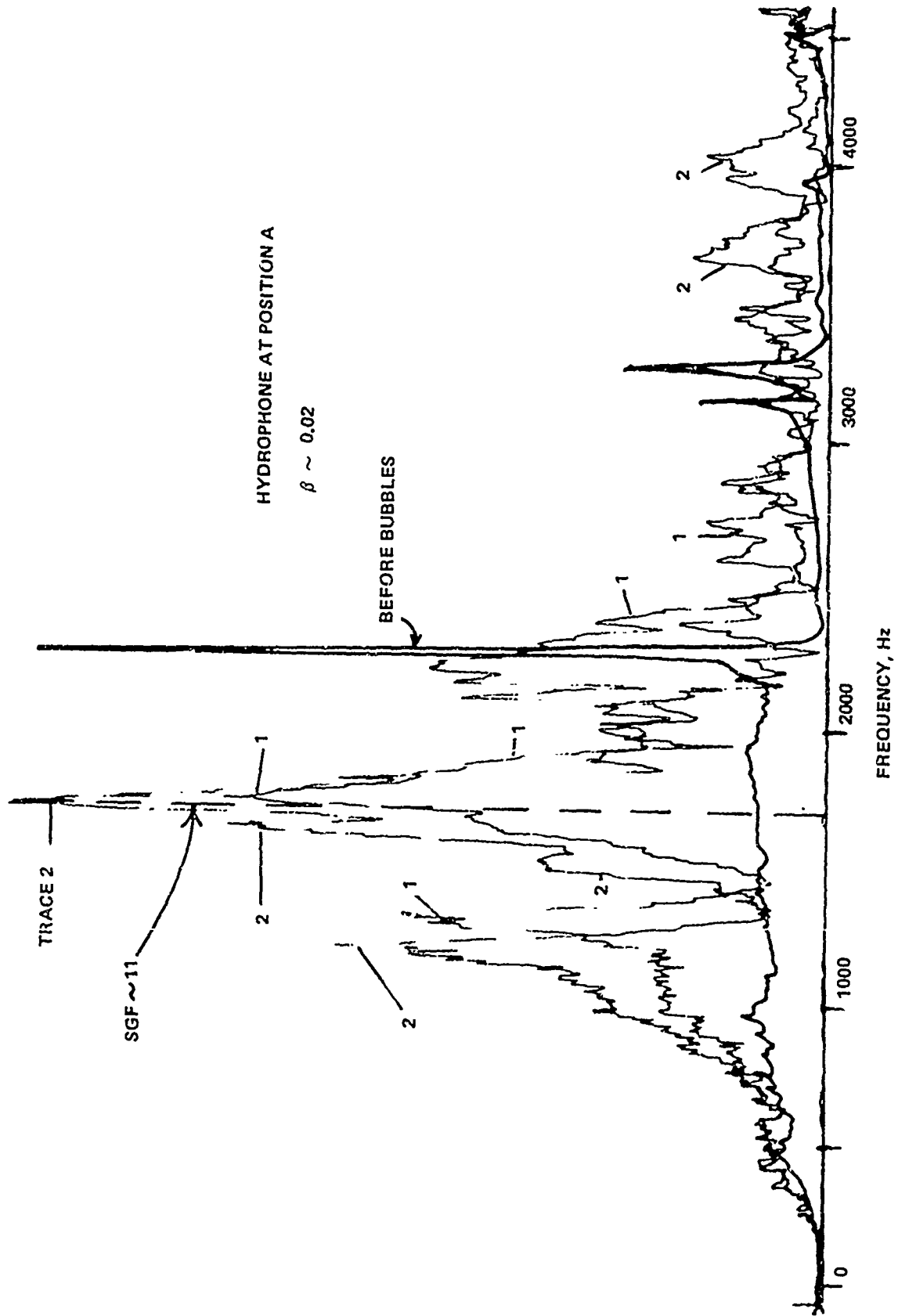


FIG. 20 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES

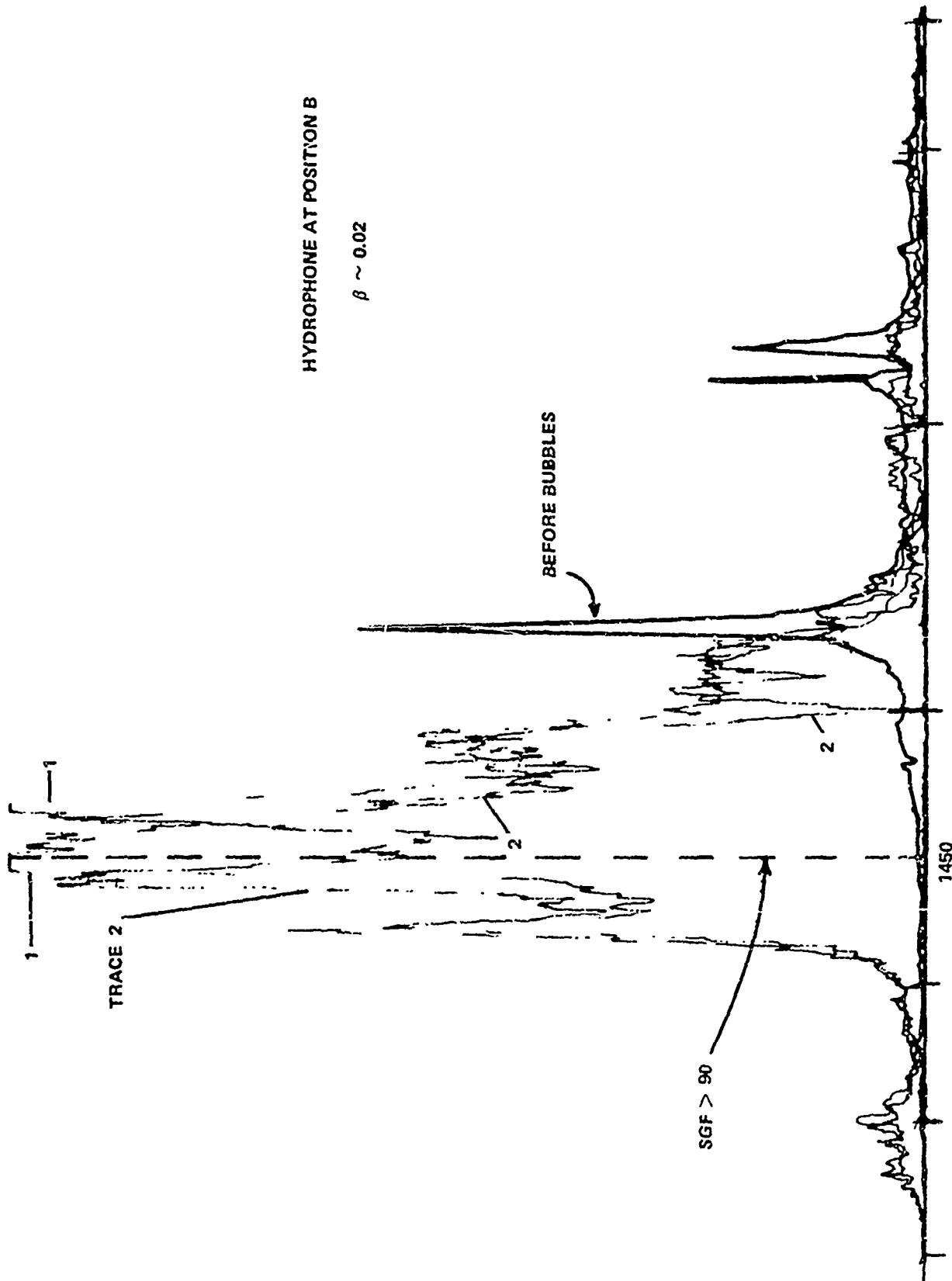


FIG. 21 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES

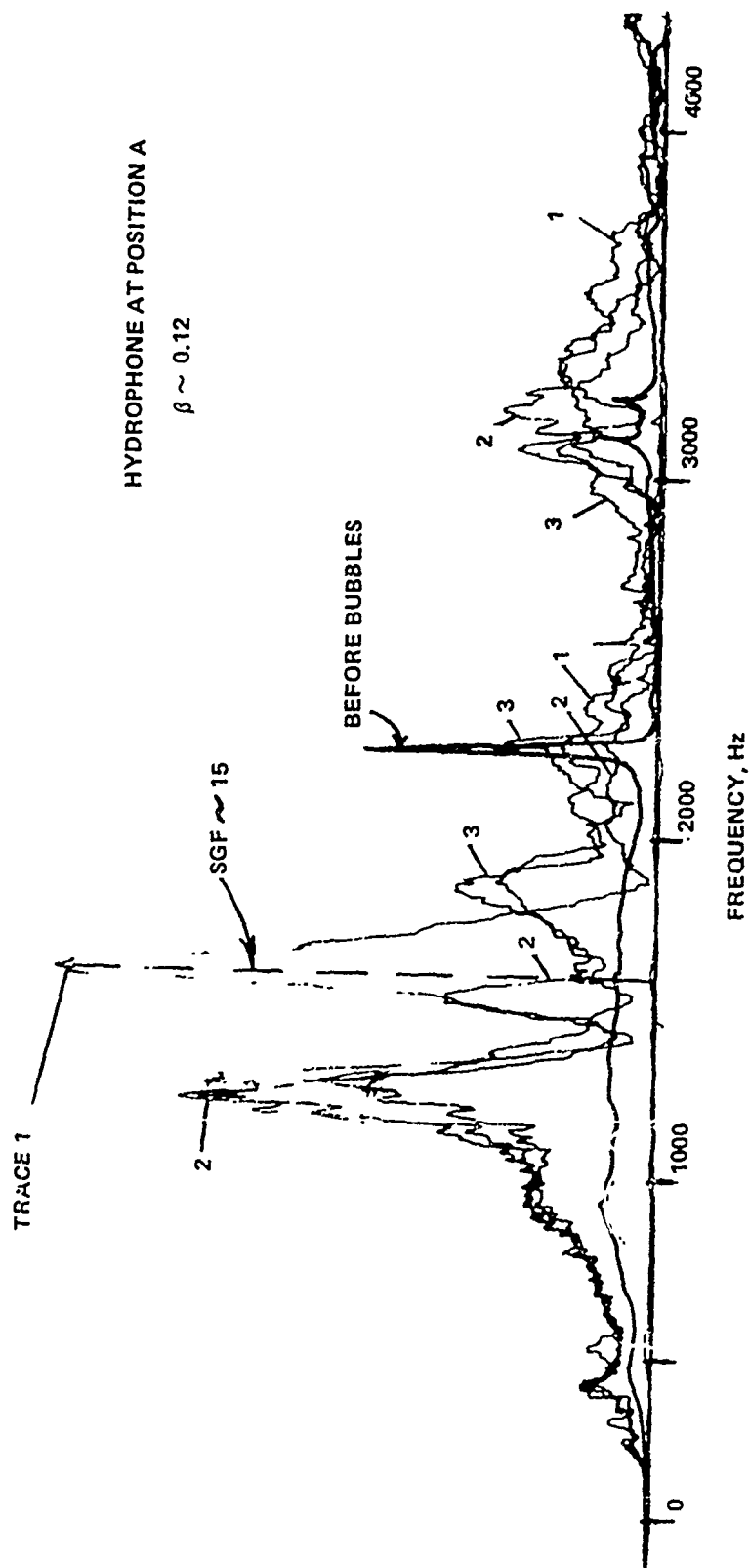


FIG. 22 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES

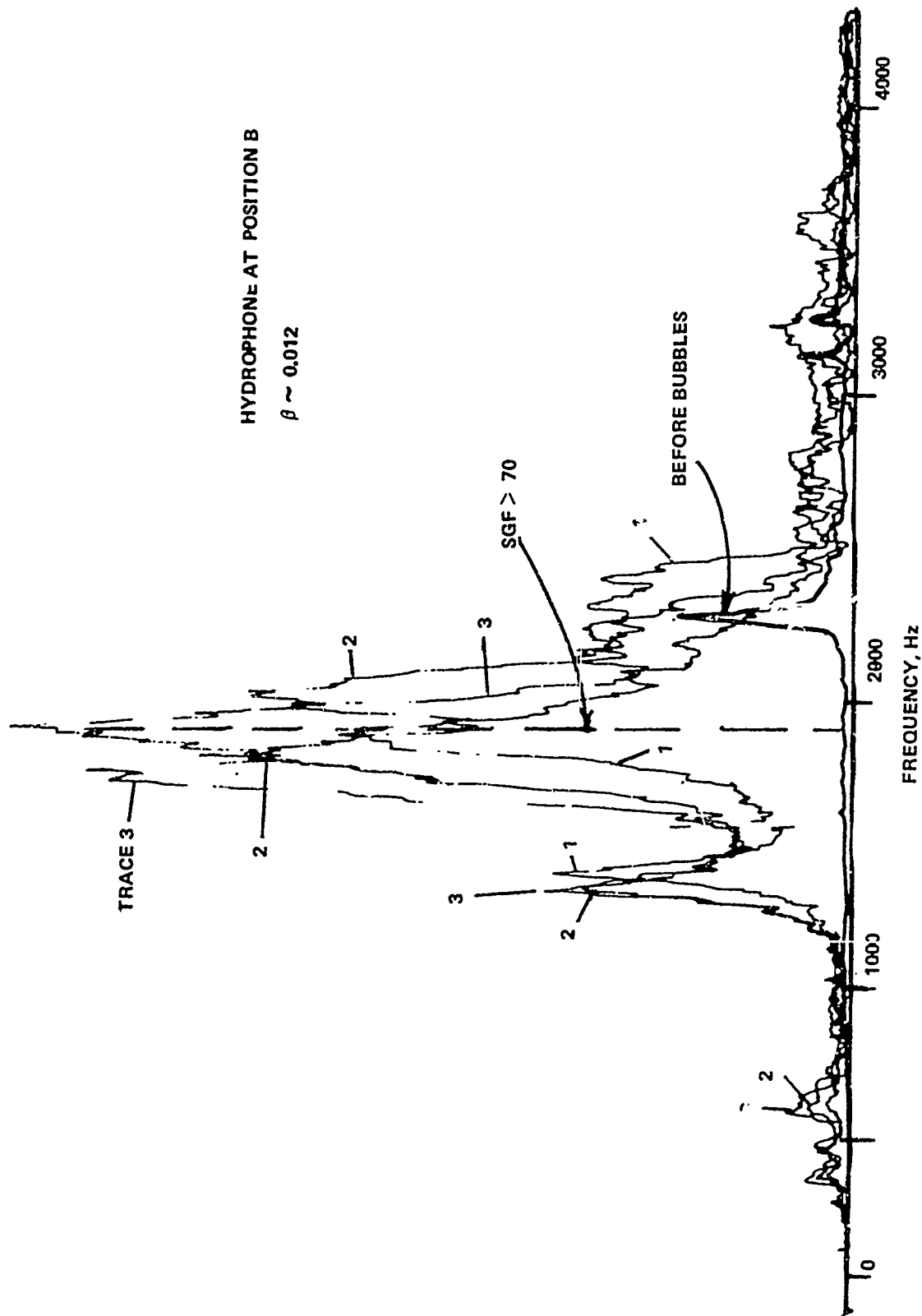


FIG. 23 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES

spectra at position B (Figures 21 and 23) we see that the maximum SGF's at position B were much greater, even though the hydrophone at position B was about five times farther from the source of the sound than it was at position A.

(e) The Dependence Of The Signal Gain Factor On Bubble Density

The simple theory that has been presented predicts that the SGF should increase with the bubble density, and this is what is found experimentally. However, a strict relation between the SGF and the bubble density in the tank has not been determined because of the difficulty in determining the bubble density in the conical bubbly region from measurements of the bubble density of the fluid that is pumped into the acoustic tank. (Nevertheless, a simple estimate has been made in Section IV.F.) The qualitative effects of increasing the bubble density are illustrated in Figures 24 through 29. Figure 19 shows the specific arrangement of the speaker, the bubble port, and the hydrophone, which was at position B for all of the measurements to be described. A comparison of figures will show that as the bubble density increases the maximum SGF increases, while the frequency of maximum SGF decreases. Figure 30 illustrates the dependence of the maximum SGF on the density of bubbles pumped into the tank, while Figure 31 illustrates the dependence of the maximum SGF frequency on the bubble density. The data in Figure 30 are nearly consistent with a simple proportionality (solid line) between the bubble density and the increase in the SGF. The data in Figure 31 may require a more complex relation. The main problem with interpreting the bubble dependences of the SGF and the maximum SGF frequency is that the average bubble densities inside the tank can only be inferred. If the flow of the bubbly water into the tank is viewed as merely a volume expansion and dilution of the flow of bubbly water in the hose, then the average bubble density within the rising column of bubbles in the tank will be essentially proportional to the bubble density in the hose (see Equation (16) in the next section). However, this simple view is complicated by the mixing of bubbly water in the column with the surrounding water and by the time dependence of the average bubble density in the tank. Apparently the water outside the bubble column, by virtue of the stirring action of the bubble flow, becomes mixed with the bubbly water, transports some of the bubbles outside the column and retains many of the very small bubbles. Thus the average bubble density inside the tank increases slowly, even though the bubble density in the hose remains essentially constant (after several minutes of stabilization). Evidence of this increase is found in the continual increases in SGF and decreases in the maximum SGF frequency as the pumping of bubbles continues (see Figures 24 through 29).

A comparison of the higher frequency (greater than 2500 Hz) pressure transmission spectra for low bubble densities with those for high densities shows that at low densities the higher frequency SGF's are often somewhat greater than one, while at high densities these SGF's are generally about unity or less than one. This could

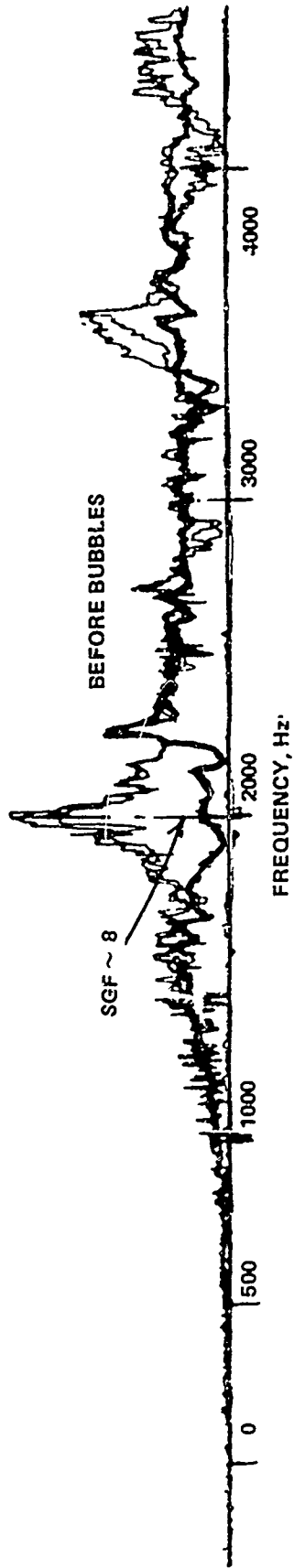


FIG. 24 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.002$

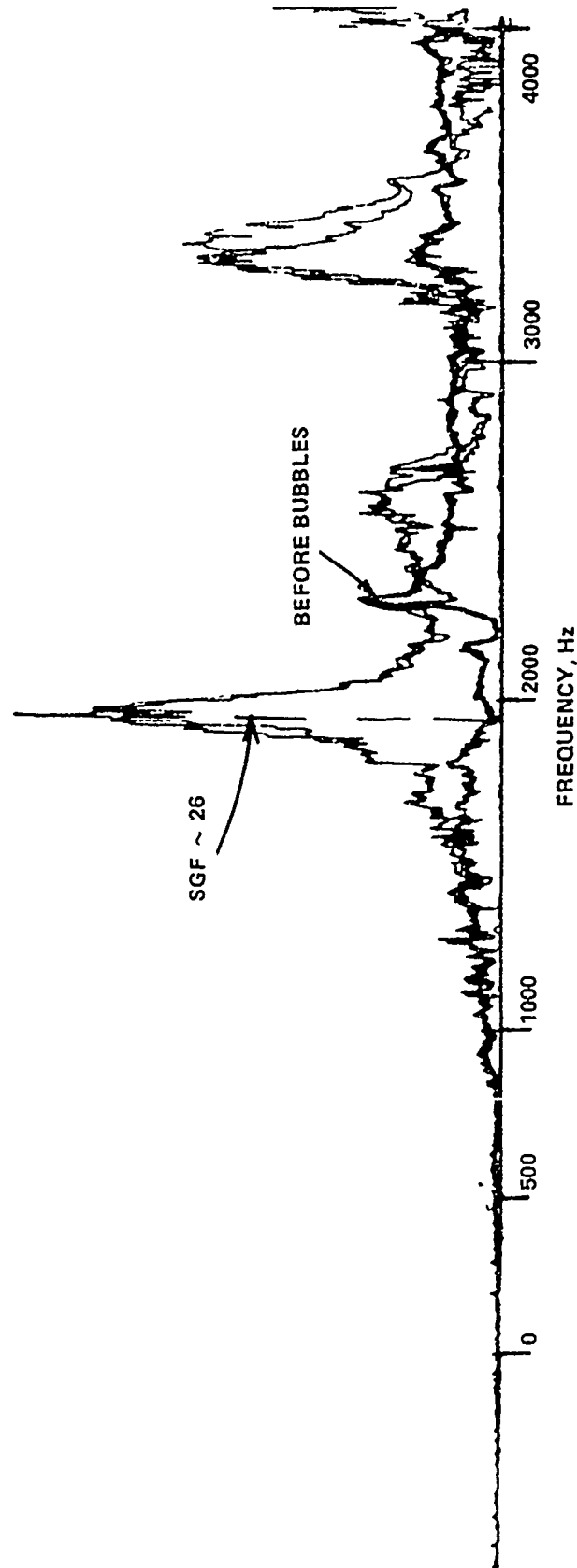


FIG. 25 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.0025$

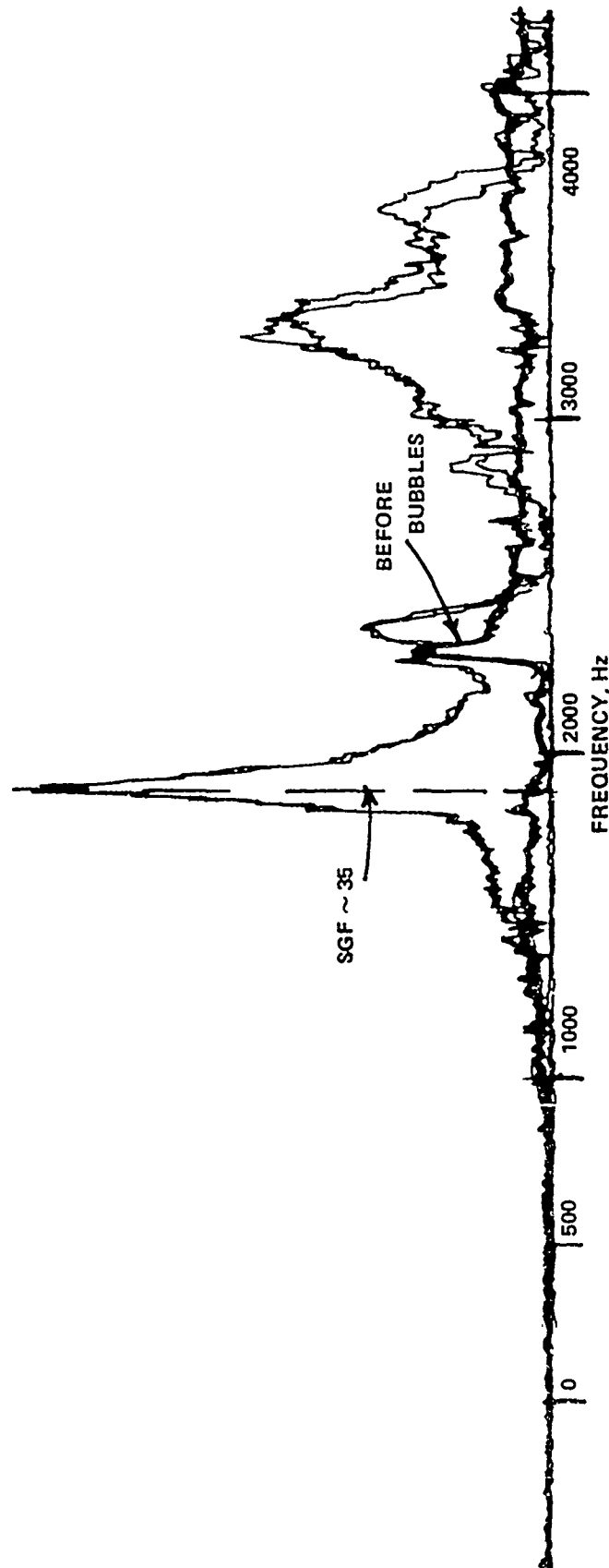


FIG. 26 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.005$

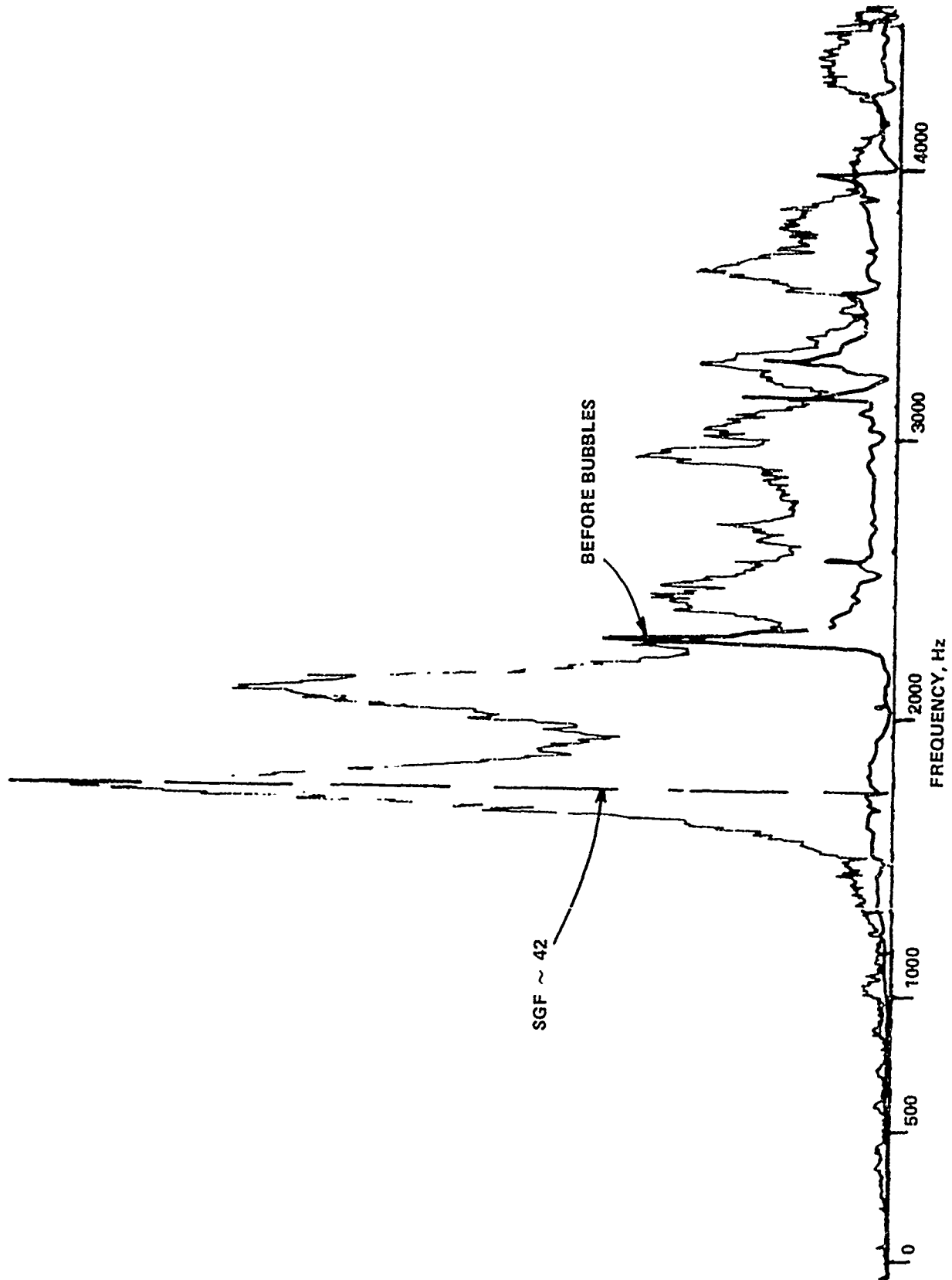


FIG. 27 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.007$

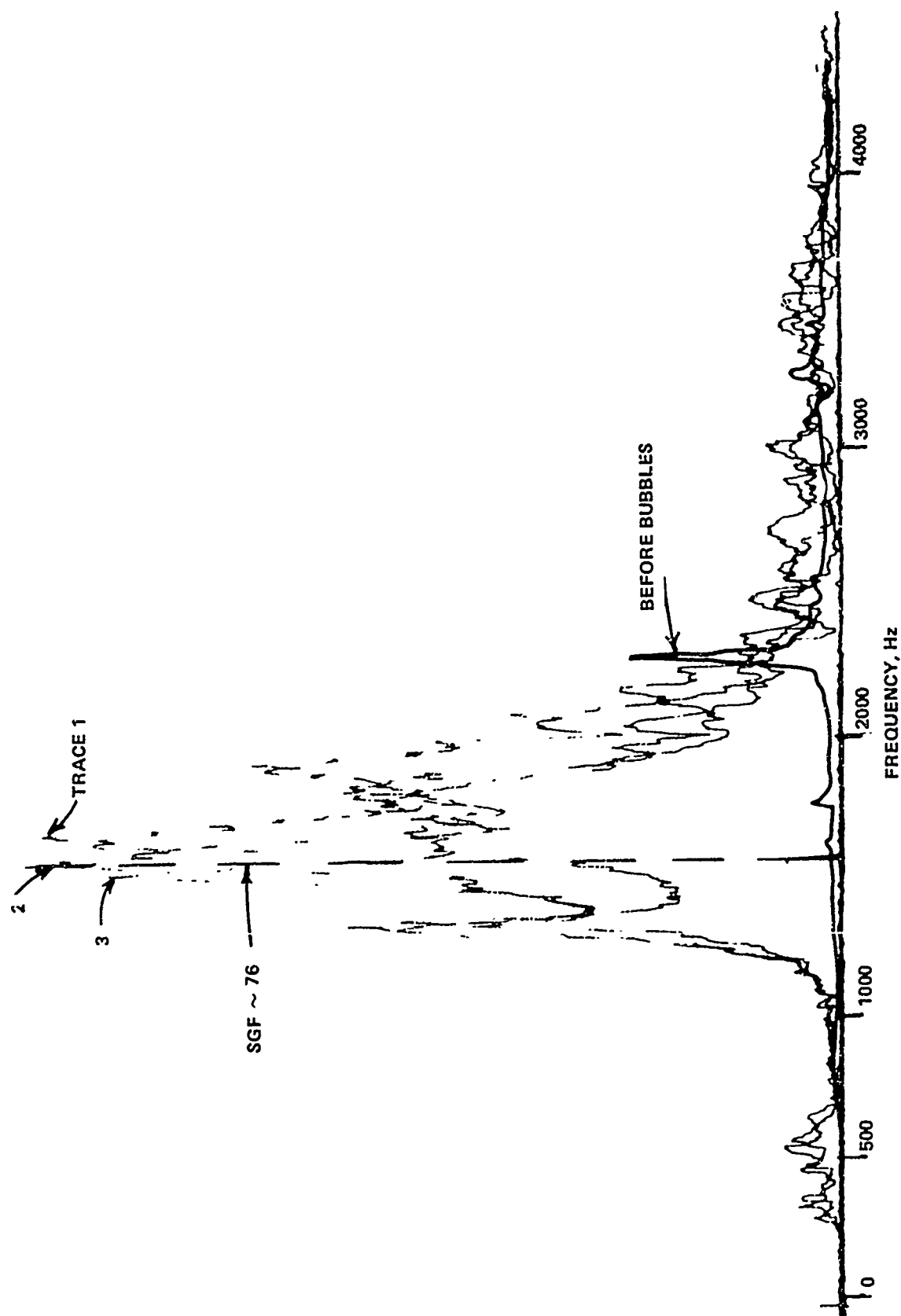


FIG. 28 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.02$

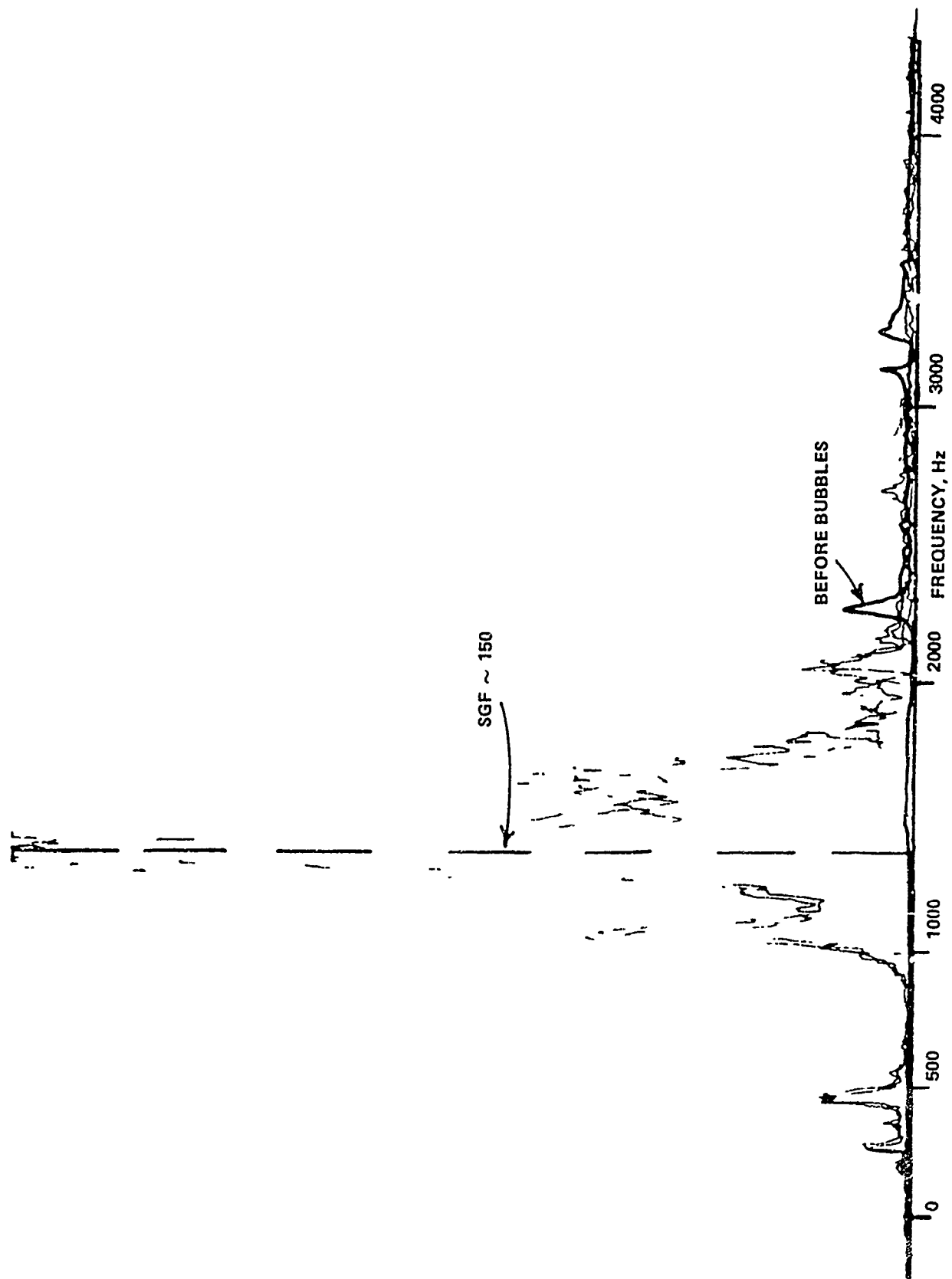


FIG. 29 ACOUSTIC PRESSURE WITH AND WITHOUT BUBBLES $\beta \sim 0.03$

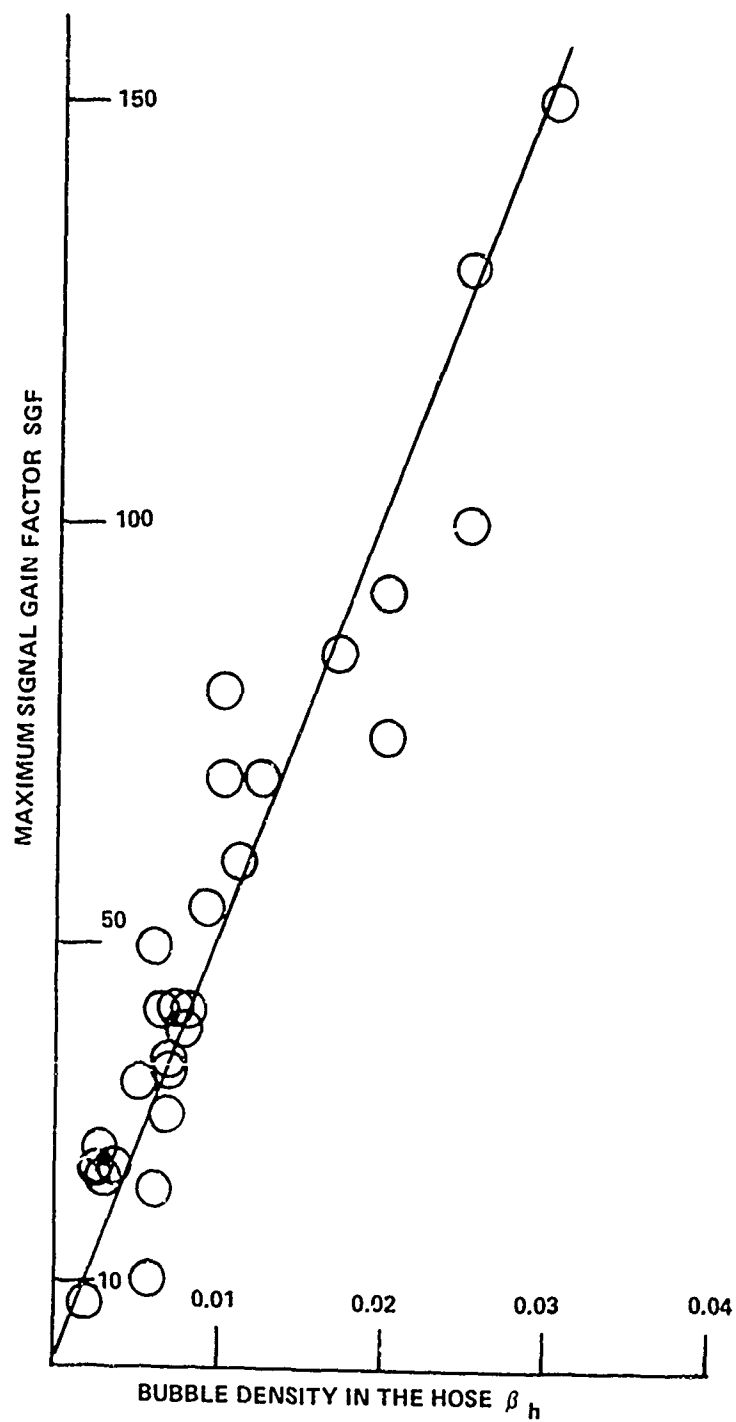


FIG. 30 THE VARIATION OF THE MAXIMUM SIGNAL GAIN FACTOR WITH BUBBLE DENSITY

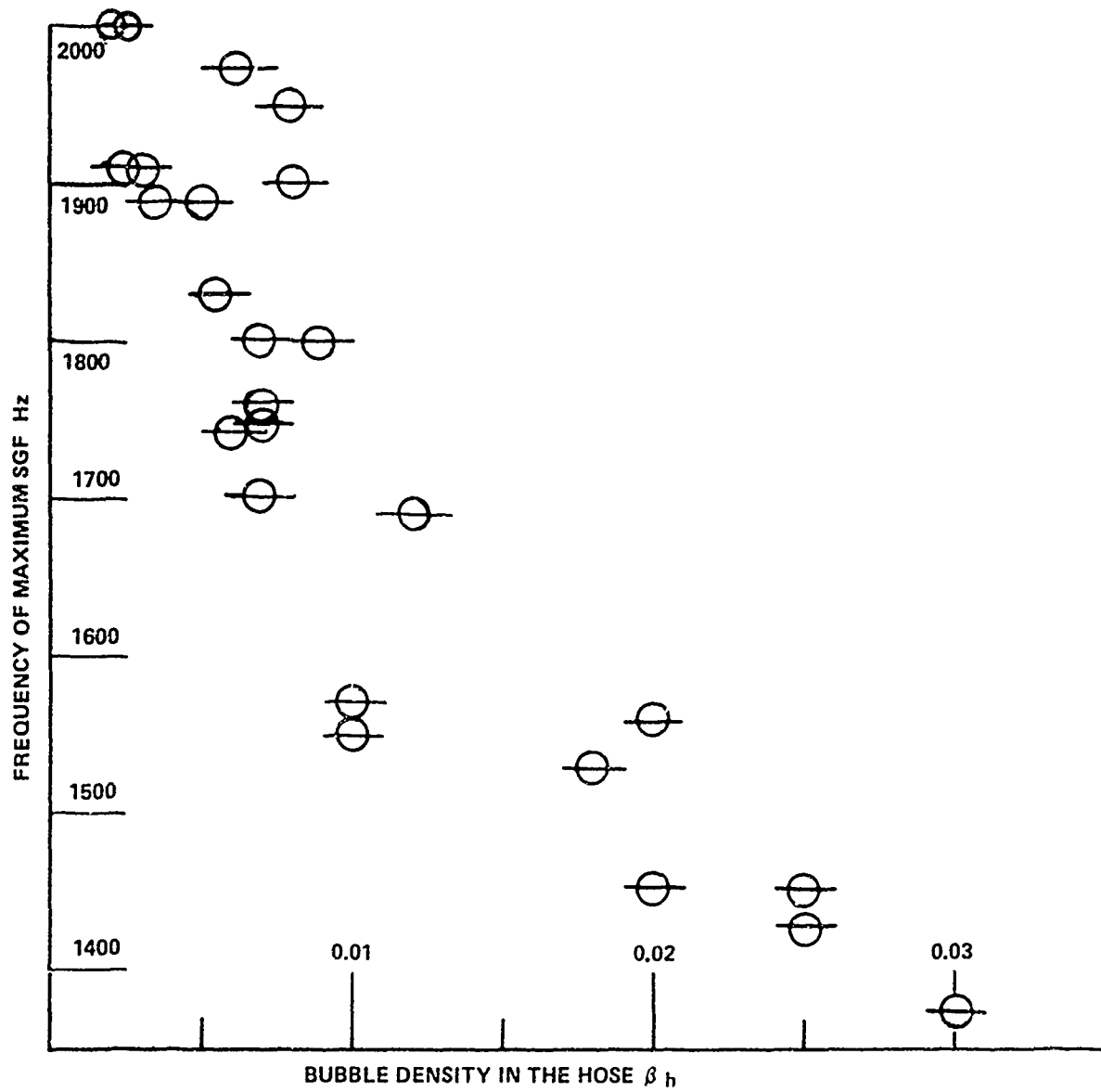


FIG. 31 THE DEPENDENCE OF THE FREQUENCY OF MAXIMUM SGF ON THE BUBBLE DENSITY

be explained if, at the large densities the sound intensity losses due to absorption and scattering, which are expected in the presence of small bubbles (see Section II), exceeds the intensity increase that results from the better impedance match to air and from sound ray trapping (if such trapping occurs). At low densities, on the other hand, the aforementioned effects which tend to increase the intensity would have to dominate over the losses due to absorption and scattering. This explanation requires that the losses due to absorption and scattering at the higher frequencies increase with bubble density at a rate greater than they do at the lower frequencies. This requirement can be met by assuming that the bubbles act as tiny inhomogeneities, the effects of which can be represented by a frequency dependent extinction coefficient. Specifically, the SGF may be phenomenologically represented as follows:

$$\text{SGF} = [1 + C_{(\beta)} G_{(\beta, f)}] e^{-L_{(\beta, f)} s} \quad (11)$$

where $C_{(\beta)}$ is an increasing function of β , G is a factor that depends upon the spatial distribution of the bubbles, the frequency, f , and the geometric arrangement of the pressure transmitter and receiver, L is an extinction coefficient that depends upon the frequency, the bubble density, and possibly upon the bubble distribution, and s is the path distance through the bubbly medium from the surface of the water to the receiver (hydrophone). The experimental results reported in this section suggest that $C_{(\beta)}$ may be approximately linear in β : $C = C_0 \beta$ (see Figure 30). If L is also proportional to β , which is a reasonable assumption since the extinction coefficient is, at least to a first approximation, proportional to the number of scattering and absorbing centers per unit volume, the signal gain factor for a particular frequency would vary as

$$\text{SGF} = [1 + C_0 \beta G_{(f, \beta)}] e^{-\beta L_{(f)} s} \quad (12)$$

For frequencies lower than $(3/4)f_r$, where f_r is the resonant frequency of the largest bubbles in the population, Equation (2) suggests that $L_{(f)}$, which depends upon the cross-section for scattering and absorption, can be replaced by

$$L_{(f)} = L_0 f^4 / (f_r^2 - f^2)^2 \quad (13)$$

since the damping constant, δ , is so small. L_0 is a constant. Equations (12) and (13) predict different SGF's for different frequencies. For example, with $f_r = 5000$ Hz (corresponding to about

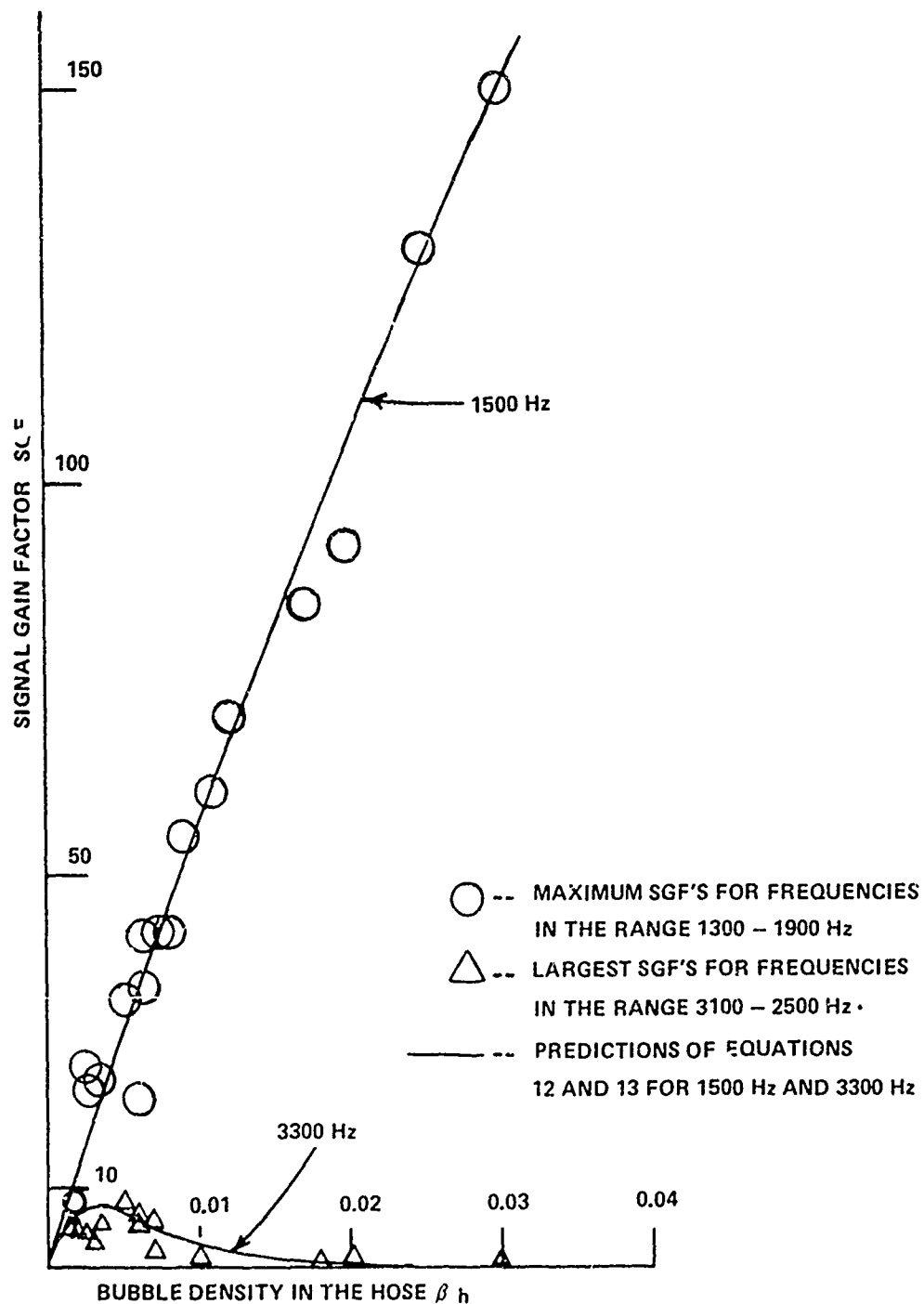


FIG. 32 THE SGF VARIATION WITH BUBBLE DENSITY AT LOW AND HIGH FREQUENCIES

0.07 cm radius bubbles in the population), $C_{OG}(\beta, f) = 5800$ (independent of β and f in this first approximation), and with $L_s = 500$, these equations predict the β dependences at 1500 Hz and 3300 Hz that are shown in Figure 32. As β increases the SGF's for the lower frequencies become much greater than for the higher frequencies because the higher frequencies are more subject to the extinction effects since they are closer to the natural bubble resonance frequency. A comparison of the spectra for various bubble densities will show that Equations (12) and (13) are in qualitative agreement with the data, as is illustrated by the data points in Figure 32.

(f) The Mode Structure Of The Bubbly Region

The spectra of Figures 24 through 27 show that when the bubble densities are low, the largest SGF's occur at several frequencies (typically about 1700 ± 200 Hz and 3300 ± 200 Hz). Thus it appears that the pressure transmission spectrum in the presence of bubbles is characterized by dominant modes, as if the bubbly region were a resonant structure similar to a waveguide. The higher frequency modes are detectable only when the bubble density is sufficiently low that the higher frequency extinction is not too great (see Equations (12) and (13) and Figure 32 for a mathematical model that represents the effects of extinction on the lower and higher frequencies of consideration here). The largest measured SGF's for various values of β occurred at the lowest frequency mode, which lay in the frequency range 1300-2000 Hz, depending upon the value of β . The higher mode frequencies are not simple multiples (harmonics) of the lowest mode frequency.

Although the bubbly region is shaped like a cornucopia, it is of interest to model it as a cylinder of essentially infinite extent (the average diameter is much less than its length) and to estimate the frequency of the lowest order mode to be expected if it were a cylindrical waveguide with rigid boundaries. In this case the dominant transmission mode frequencies are given by the zeroes of the zeroth order Bessel function as follows:

$$f_n = (Z_n/\pi)(v/2R) \quad (14a)$$

$$J_0(Z_n) = 0 \quad (14b)$$

where Z_n is the value of the argument of the Bessel function, $J_0(Z_n)$, v is the sound velocity in the waveguide, and R is the radius of the waveguide. The bubbly region can be approximated as a cylinder of average diameter about 5 cm. The sound velocities depend upon the value of β_b , the density in the bubbly region, according to the relation expressed by Figure 2 (Equation (3)). (For $0.001 < \beta \leq 0.05$, $v \approx 10/(\beta)^{1/2}$ m/sec.) However, the average value of β_b is not equal

to the value β_h that has been measured in the bubble hose, but rather is somewhat less than that value. Estimates of the average flow velocities in the bubble hose and in the bubbly region suggest that the two velocities were, respectively, about 64 cm/sec and 12-15 cm/sec. Assuming that the bubbles are "conserved", then the number of bubbles flowing through any cross-sectional area within the bubble hose and also through any cross-sectional area within the bubbly region (one dimensional flow) is given by

$$dN/dt = \beta A v = \text{a constant} \quad (15)$$

where N is the number of bubbles flowing through the cross-sectional area, A . Inside the hose $A = 0.28 \text{ cm}^2$ and $v = 64 \text{ cm/sec}$. Therefore, within the bubbly region, represented as a cylinder of cross-sectional area equal to about 20 cm^2 and with the velocity in the bubbly region 12-15 cm/sec.,

$$\beta_b \approx 0.06\beta_h \text{ to } 0.075\beta_h \quad (16)$$

This relation allows a reasonable estimate of β_b . Equation (14) can now be written

$$f_n \approx (Z_n/16)v(\beta_b) \approx (Z_n/16)(1000/\sqrt{\beta_b}) \quad (17)$$

where v is in cm/sec. The lowest order zeroes of the Bessel function are $Z_1 = 2.405$ and $Z_2 = 5.52$. Equation (17) predicts that when $\beta_h = 0.03$, which corresponds to $\beta_b \approx 0.0018 - 0.0022$, and $v \approx 240-220 \text{ m/sec}$.

(from Figure 2), the lowest order mode frequencies are in the ranges 3600-3300 Hz and 8300-7600 Hz. These frequencies are too high by a factor of about 2.5. However, the numerical disagreement is not unexpected since the calculation of the mode frequencies was based upon an extremely simplified model. It is probable that a more complete theoretical calculation based on a conical model with diffuse boundaries will yield lowest order waveguide modes that are somewhat lower in frequency than the values obtained here.

For low values of β this model is in even worse disagreement with the data. When $\beta_h = 0.0025$, $\beta_b = 0.00015-0.0001875$ and $v = 730 \text{ m/sec} - 670 \text{ m/sec}$. The lowest order mode frequency is now predicted to be in the range 11,000-10,000 Hz, which is more than 5 times greater than the measured frequency (Figure 25) of about 1900 Hz. Whether an improved waveguide model could overcome this amount of discrepancy is at present an open question.

V. DISCUSSION

The preceding experimental results show that a column of bubbles in a tank noticeably increases the sound pressure at certain frequencies and at certain places in the tank providing that the bubble density is sufficiently large. Exactly how much of this effect is due to enhanced air-water sound transmission plus, perhaps, sound trapping in the bubbly medium, and how much is due to modification of the mode structure of the tank is not at present known. In order to be absolutely sure of the effects of the bubbles alone it would be necessary to repeat the experiments in either a totally damped tank or in an "infinite" tank (a lake, etc.). In the latter case reflections from distant surfaces could be made experimentally unimportant by using acoustic pulses of durations comparable to $(1/f)$, where f is the frequency to be transmitted from air into the water. The lowest frequency that could be tested in this way would be $(1500/R)$, where R is the range in meters from the bubbly region to the nearest reflective surface.

In light of the results of the simple theory presented in the second section, it is not surprising that tiny bubbles increased the sound pressure by an amount that depended upon the concentration and distribution of the bubbles. However, the largest expected SGF's (30 for a point source in air radiating into a bubbly layer and 60 for a plane wave traversing a plane bubbly layer with perfect matching between air and water impedances (see Figures 6 and 7)) were expected to occur when bubble densities were greater than 0.1. Yet SGF's greater than 100 have been observed at particular frequencies in some experiments, even though the average density in the bubble column has probably never exceeded 0.005. Such large gains were even observed in the few instances when the (small) tank was quite mode free (Figure 16) which suggests that these large gains really are due to the bubbles and not to rearrangements of the mode structure of the tank. Assuming then, that they are due to the bubbles one must ask how it is possible for the bubbles to be so "efficient"; that is, to produce very large SGF's with less than the expected optimum bubble densities.

The answer that is proposed has to do with the actual geometry of the bubbly region that has been used in these experiments. The geometry is too complicated to be treated in a simple manner so it is not yet possible to offer a quantitative understanding of the effects of the bubbles. However the basic characteristics of conical bubbly columns can be qualitatively understood. The suggested answer

is that the bubbly region partially traps and guides a portion of the sound pressure waves that are incident on the water where the bubble column reaches the surface. The bubble column can trap sound rays because it is a medium of higher refractive index (lower sound velocity) than the surrounding bubble-free water. If the high index region were very large compared to the wavelength of the sound, the sound rays would be bent into the high index region. Figure 33A illustrates this effect in the case of two adjacent media with an infinitely thin boundary between them. There is a critical angle such that for all angles (defined with respect to the normal to the boundary as in Figure 33A) greater than the critical angle, sound rays are reflected back into the medium of higher refractive index. The critical angle is given by Snell's law (Equation (5)) as

$$\sin \theta_c = n_2/n_1 = v_1/v_2 \quad (11)$$

where the n 's are the refractive indices of the two media. From Figure 2, for $\beta = 10^{-5}$, $v_1/v_2 = 1380/1520$ and $\theta_c \sim 65^\circ$; for $\beta = 10^{-4}$, $\theta_c \approx 34^\circ$; for $\beta = 10^{-3}$, $\theta_c \approx 12^\circ$, and for $\beta = 10^{-2}$, $\theta_c \approx 4^\circ$. Thus for the large values of β used in this experiment almost all the sound rays incident on the boundary would be trapped. In the more realistic case of a uniform variation in refractive index from the bubbly medium to the bubble-free water (Figure 33B), the sound rays are bent continuously as if by an infinite number of infinitely thin layers of medium that differ in refractive index by infinitesimal amounts. If the actual refractive index variation with distance were known, either as an analytic expression or as a table of numbers vs. distance, a computer could calculate the sound ray trajectory by a series of successive applications of Snell's law. The qualitative result would be the same as for the case of a sharp boundary between the media. For the particular case of a bubble distribution of cylindrical shape the result of the calculation would show that sound rays which travel along the cylinder at angles that are greater than some critical angle will be trapped within the cylinder, as in Figure 33C. By using a particular form of a ray equation (which incorporates Snell's law) with a Gaussian radial distribution of bubbles, King²¹ showed that sound rays would be trapped in the nearly cylindrical wake of a propeller. He found that for $\beta \approx 3 \times 10^{-4}$, trapping occurred for θ at least as small as 30° (the application of Snell's law at a sharp boundary predicts trapping for θ as small as 22°).

Since the bubbly region is actually conical rather than cylindrical the sound waves which are trapped at the wide base of the cone (the air-bubbly water surface) would be concentrated as they travel

²¹W.F. King, III, J.A.S.A. 54, 735 (1973)

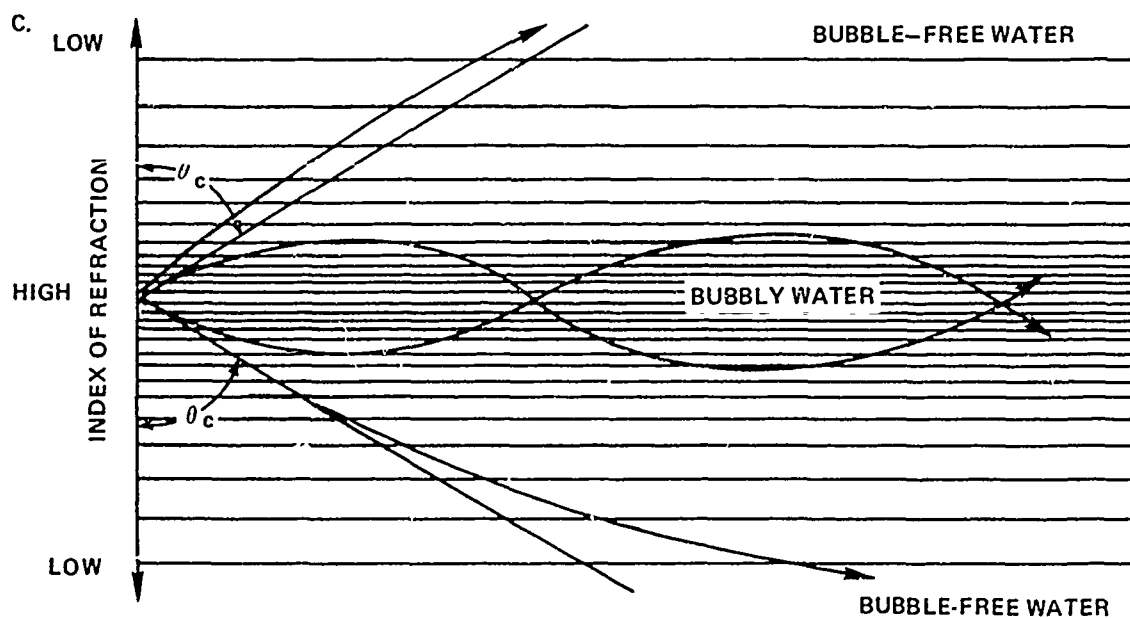
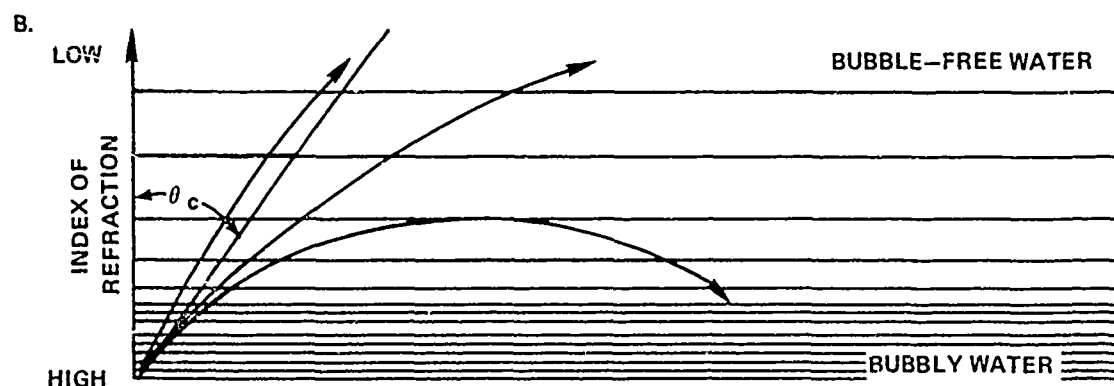
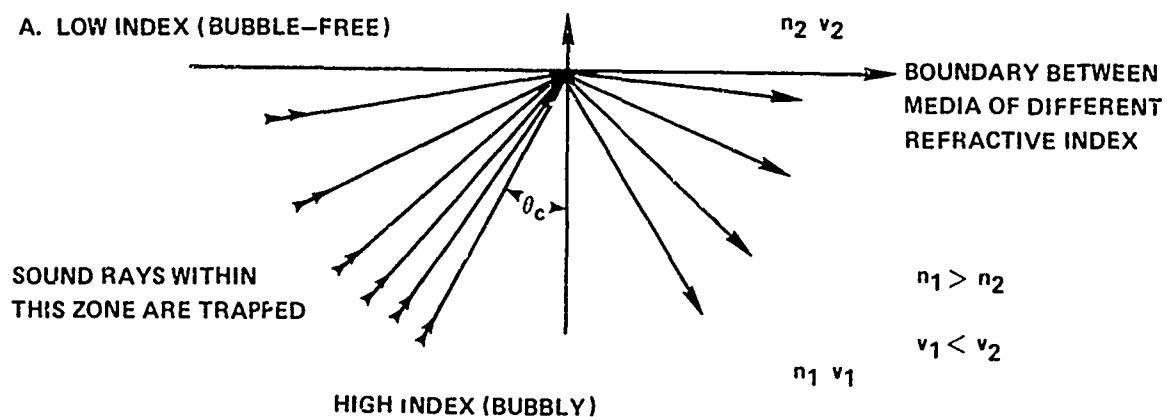


FIG. 33 SOUND RAY TRAPPING

toward the apex of the cone at the bubble port. This could explain the large increases in sound pressure.

Unfortunately the simple explanation just given for the large sound pressures does not account for the mode structure that seems to be characteristic of the transmission spectrum when the bubbles are present (three modes - about 2000 Hz, about 2700 Hz, and about 3300 Hz when β is very small, and a single mode - less than 1800 Hz - when β is large). The reason that the previous explanation does not account for mode structure is that the only requirement on the wavelength is that it be much smaller than a characteristic dimension, such as the diameter, of the region of high refractive index (the fundamental ray optics condition). All wavelengths which meet this condition should be trapped. A theoretical reason for not using this model to describe the bubble experiments reported here is that the fundamental ray optics condition is violated by the experimental conditions: the diameter of the bubbly region (about 10-15 cm at the water surface to less than a centimeter at the bubble port) is comparable to or smaller than the wavelengths within the bubbly region at the frequencies which have exhibited large SGF's (between 15 and 30 cm at the lowest frequencies and 7 to 15 cm at the highest frequencies). An alternative model treats the bubbly region as a waveguide with natural resonant mode frequencies of transmission that depend upon the geometry of the region. Sound waves at the natural frequencies of the waveguide are transmitted with small loss along the guide. Numerical results of treating the region as a cylindrical waveguide are presented in Section IV.F. However, the results make it clear that a simple waveguide model of the bubbly region is not sufficient.

The experimental results which prefer the waveguide model over a ray optics trapping model are the spectra which clearly show a mode structure of the signal gain in the presence of bubbles. However, these spectra may not be conclusive with respect to requiring a waveguide model because the mode structure with bubbles may quite closely be related to the mode structure without bubbles. As stated before, in order to be absolutely sure that the mode structure with bubbles is truly a characteristic of the shape, density, etc., of the bubble region, these experiments must be repeated in a substantially mode free acoustic tank.

VI. CONCLUSION

Dense concentrations of bubbles at the surface of a volume of water can affect the transmission of sound from air into the water by decreasing the impedance difference between air and (bubbly) water and thereby reducing the sound reflection at the water surface. Sound rays incident on the surface of bubble free water are totally reflected if the angle of incidence (with respect to the normal) is greater than 13° . However, dense bubble populations can increase the angle of total reflectance up to 90° so that sound rays incident at essentially all angles can be transmitted into the water. The

conical bubble distributions used in these experiments increased the underwater sound pressure by as much as two orders of magnitude at certain frequencies. Although some of the measured increase (signal gain factor) may have resulted from shifting of the resonant modes of the water tanks, it is believed that at least a major portion of the measured SGF was a direct result of the presence of the bubbles. The band of frequencies that was most enhanced by the bubbles depended on the bubble density. For densities below 0.01, as measured in the bubble hose, the band was about 500 Hz wide and was centered around 2000 Hz. For larger densities the band center moved downward in frequency. The reason for preferred bands of frequencies is not known. It may be that the bubble distribution acts like a waveguide, but only experiments with different bubble distributions can provide a conclusive answer to this question and to the question of why the "preferred" frequency depends upon the bubble density. Further experimentation to answer these questions should be done in mode-free containers, i.e., tanks which are very large and/or acoustically damped, and an improved bubble making device which would allow some modification of the shape of the bubbly region should be used.